CHAPTER 1: THE BASIC INTERTEMPORAL MODEL

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Topics to be covered

- Differences between closed and open economies.
- Extension of the two period problem to an infinite horizon setting in continuous time.
  - Study the solution and the response of the economy to temporary and permanent shocks.
- Add investment to the basic model
  - Study the response of the economy to changes in productivity.

1. Introduction

- What is the fundamental difference between a closed and an open economy?
  - A closed economy is forced to consume and invest within its own current resources (investment $\leq$ savings).
  - An open economy can borrow from the rest of the world, thus it can engage in intertemporal trade to smooth consumption and finance investment within its own lifetime resources.

2. The Model

- Small open economy inhabited by a large number of identical, infinitely-lived consumers.
- No uncertainty, perfect foresight.
- Only one good (tradable and non-storable).
- Small open economy: takes price of tradable good as given.
- The economy is endowed with a flow of the good (*i.e.* no production).
- No government.
- Perfect capital mobility in the sense that consumers can borrow/lend in international capital markets at a constant real interest rate, r.

2.1 Consumer's Problem

2.1.1 Preferences

Let c_t denote consumption. The objective is to

$$\max \; U(c_t) = \int_0^T u(c_t) e^{-\beta t} dt \tag{1}$$

with $\quad \beta > 0, \qquad u' > 0, \qquad u'' < 0, \qquad \lim_{c_t \to 0} u'(c_t) = \infty \qquad (2)$

2.1.2 Flow constraint

Let b_t denote net foreign assets.

$$\dot{b}_t = r b_t + y_t - c_t \tag{3}$$

with b_0 given, $b_T \geq 0$.

2.1.3 Basic Formulation

The consumer's maximization problem can be formally stated as :

$$\max_{\{c_t\}_{t=0}^{T}} U(c_t) = \int_0^T u(c_t) e^{-\beta t} dt$$

subject to

$$\begin{aligned} &\dot{b}_t = r b_t + y_t - c_t \\ &b_T \geq 0 \\ &b_0 \text{ given} \end{aligned} \qquad (4)$$

Given $u'(c_t) > 0 \rightarrow$ no satiation point $\rightarrow$ at the optimum $b_T \leq 0$.

$$b_T = 0 \qquad (5)$$

Solving this model

Note that equation (1) is an integral (NPDV of utility), while (3) is a differential equation (evolution of assets). To solve this problem we would need calculus of variations as we are dealing with the maximization of quantities that depend on functions.

We could transform the problem to solve it with standard Lagrange multiplier techniques. To this end, we reformulate the problem from an uncontable number of constraints (i.e. flow constraints in each point in time) into a maximization problem subject to one constraint (i.e. intertemporal constraint).

2.1.4 Intertemporal Budget Constraint

Rewrite $\dot{b}_t - rb_t = y_t - c_t$ (3) in present value

$$\left(\dot{b}_t - rb_t\right)e^{-rt} = \left(y_t - c_t\right)e^{-rt} \tag{6}$$

Integrate forward

$$\int_0^T \left(\dot{b}_t - rb_t\right)e^{-rt}dt = \int_0^T \left(y_t - c_t\right)e^{-rt}dt \tag{7}$$

Notice that the LHS of (7) can be solved to yield

$$\int_0^T \left(\dot{b}_t - rb_t\right)e^{-rt}dt = \int_0^T \tfrac{d\left(b_t e^{-rt}\right)}{dt}dt = e^{-rT}b_T - b_0 \tag{8}$$

where we used the result that for a variable x_t and constants a and b the differential equation $\dot{x}_t + ax_t = b$ has the solution $x_t = Ce^{-at} + \frac{b}{a}$ to derive $\dot{b}_t - rb_t = 0 \Leftrightarrow b_t = Ce^{rt}$ so $\left(b_t - Ce^{rt}\right)e^{-rt} = b_t e^{-rt} - C.$

Intertemporal Budget Constraint (cont.)

Substituting (8) into (7) yields

$$e^{-rT} b_T - b_0 = \int_0^T (y_t - c_t) e^{-rt} dt \qquad (9)$$

Taking into account $b_T = 0$ from (5) we can rewrite the consumer's intertemporal budget constraint (9) as

$$\underbrace{b_0 + \int_0^T y_t e^{-rt} dt}_{\substack{\text{NPDV of} \\ \text{life-time resources}}} = \underbrace{\int_0^T c_t e^{-rt} dt}_{\substack{\text{NPDV of} \\ \text{life-time consumption}}} \qquad (10)$$

2.1.5 Alternative Formulation

Using this intertemporal budget constraint we can reformulate the consumer's maximization problem as :

$$\max_{\{c_t\}_{t=0}^{T}} U(c_t) = \int_0^T u(c_t) e^{-\beta t} dt \qquad (1)$$

subject to

$$b_0 + \int_0^T y_t e^{-rt} dt = \int_0^T c_t e^{-rt} dt \qquad (10)$$

This problem can be expressed using a Lagrangian as :

$$\max_{\{c_t, \lambda\}_{t=0}^{T}} L = \int_0^T u(c_t) e^{-\beta t} dt + \lambda \left(b_0 + \int_0^T y_t e^{-rt} dt - \int_0^T c_t e^{-rt} dt \right)$$

Alternative Formulation (cont.)

$$\max_{\{c_t,\lambda\}_{t=0}^{T}} \; L = \int_0^T u(c_t)e^{-\beta t}dt + \lambda\left(b_0 + \int_0^T y_t e^{-rt}dt - \int_0^T c_t e^{-rt}dt \right)$$

First order necessary conditions (FONC) for optimization include:

$$\frac{\partial L}{\partial c_t} = 0 \quad \Rightarrow \quad u'(c_t)e^{-\beta t} = \lambda e^{-rt} \quad \forall t = 1, \ldots, T \qquad (11)$$

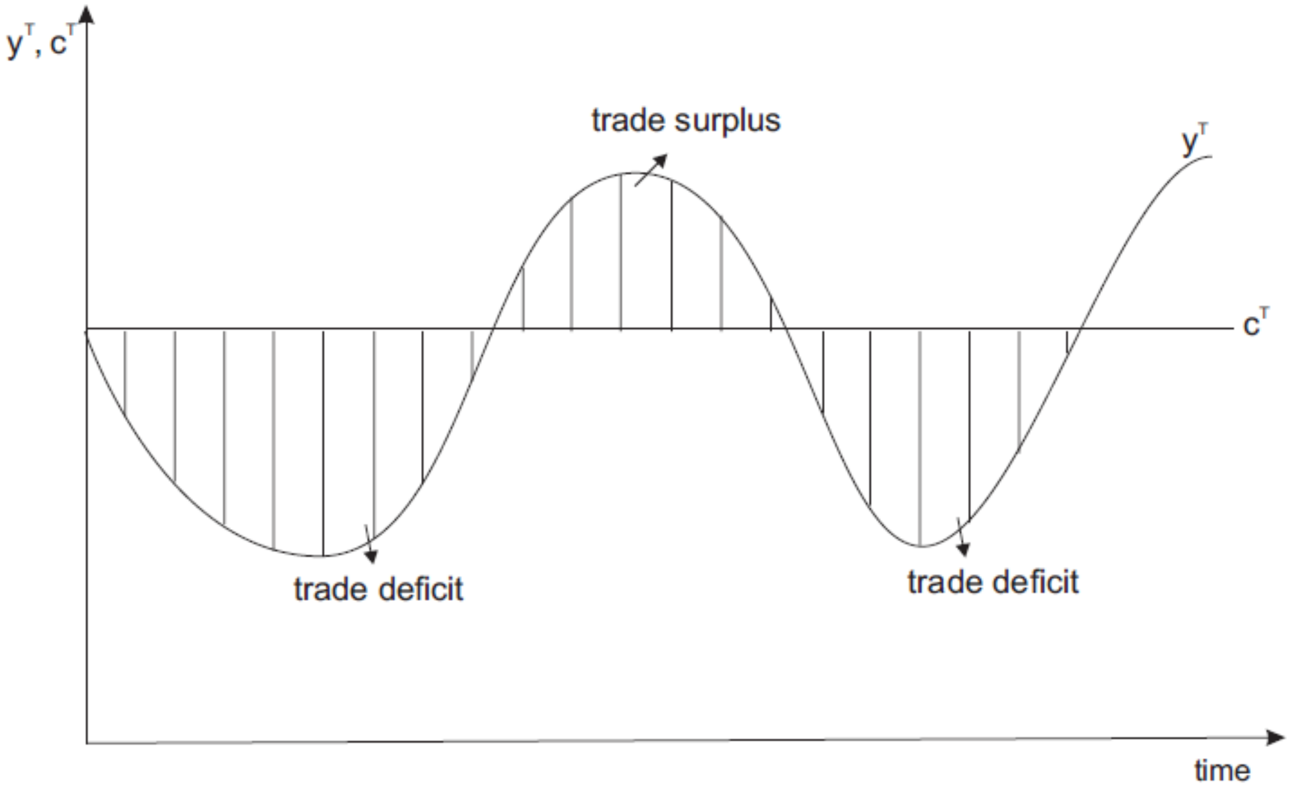
$$\frac{\partial L}{\partial \lambda} = 0 \quad \Rightarrow \quad (10)$$

Furthermore, if we assume perfect capital markets $(\beta = r)$, FONC (11) simplifies to

$$\underbrace{u'(c_t)}_{\text{Marginal Utility of Consumption}} = \underbrace{\lambda}_{\text{Marginal Utility of Wealth}} \qquad (12)$$

Consumption and Output over Time

Figure 1. Perfect foresight paths



2.2 Infinite Horizon Formulation

In this case the consumer's maximization problem becomes:

$$\max_{\{c_t\}_{t=0}^{\infty}} V(c_t) = \int_0^{\infty} u(c_t) e^{-\beta t} dt$$

subject to

$$\dot{b}_t = r b_t + y_t - c_t$$

$$\lim_{t \to \infty} e^{-rt} b_t \geq 0 \qquad (14)$$

$$b_0 \text{ given}$$

Note that the no-Ponzi games condition (14) requires that the present discounted value of debt "at the end of the consumer's life" should be non-negative. So, we are not using

$$\lim_{t \to \infty} b_t \geq 0 \qquad (15)$$

Infinite Horizon Formulation (cont.)

Note that $\lim_{t\to\infty} e^{-rt} b_t \geq 0$ (14) is weaker than $\lim_{t\to\infty} b_t \geq 0$ (15). In particular, note that asymptotic debt (*i.e.* $\lim_{t\to\infty} b_t < 0$) is consistent with (14), which only imposes a constraint on the growth rate of debt (less than r).

As before, the non-satiation assumption ensures that the consumer will never want to "die" with a positive level of assets. Therefore, optimality requires

$$\lim_{t\to\infty} e^{-rt} b_t \leq 0 \qquad (16)$$

The combination of the no-Ponzi games condition (14) and optimality condition (16) gives the transversality condition

$$\lim_{t\to\infty} e^{-rt} b_t = 0 \qquad (17)$$

Infinite Horizon Formulation (cont.)

By following the same series of steps as above and using the transversality condition (17) we can derive the infinite horizon intertemporal constraint:

$$b_0 + \int_0^{\infty} y_t e^{-rt} dt = \int_0^{\infty} c_t e^{-rt} dt \qquad (18)$$

which can be used to re-state this maximization problem in terms of a Lagrangean:

$$\max_{\{c_t, \lambda\}_{t=0}^{\infty}} L = \int_0^{\infty} u(c_t) e^{-\beta t} dt + \lambda \left(b_0 + \int_0^{\infty} y_t e^{-rt} dt - \int_0^{\infty} c_t e^{-rt} dt \right)$$

The FONC with $T \to \infty$ continue to be $u'(c_t) e^{-\beta t} = \lambda e^{-rt}$ (11) and (18).

From now on we will work with the infinite horizon model.

2.3 Interpretation of the Lagrange Multiplier

- In micro theory λ is usually described as the shadow price of wealth.
- This interpretation is valid only if the path of intertemporal relative prices is constant. More generally, λ will depend not only on wealth, but also on the path of intertemporal relative prices (see chapter 3).
- We can also think of $\frac{\lambda}{r}$ as an asset price and of λ as a dividend.

Assume $\beta = r$ such that FONC (11) reduces to $u'(c_t) = \lambda$ (12).

Multiply both sides by e^{-rt} and integrate forward to obtain

$$\frac{\lambda}{r} = \int_0^\infty u'(c_t) e^{-rt} dt$$

Then $\frac{\lambda}{r}$ is the price of an asset that would offer the consumer a "dividend" of $u'(c_t)$ at every point in time. When $u'(c_t)$ is constant λ is also the dividend. As a price, λ should only change in response of unanticipated information.

2.4 Equilibrium Conditions

Since the consumer is the only agent in this economy, the flow constraint (3) and the intertemporal constraint (18) are also the economy's aggregate constraints.

Let TB_t trade balance be given by net exports of goods and services

$$TB_t \equiv y_t - c_t \qquad (19)$$

and IB_t income balance be given by net factor payments from abroad

$$IB_t \equiv rb_t \qquad (20)$$

Also by definition, the current account is given by :

$$CA_t \equiv TB_t + IB_t \qquad (21)$$

Substituting (21) into (3), it follows that :

$$\dot{b}_t = CA_t \qquad (22)$$

Equilibrium Conditions (cont.)

Define the capital account balance, KA, in the standard way (whereby acquiring claims against the rest of the world implies a negative capital account balance), we have:

$$KA_t \equiv -\dot{b}_t \qquad (23)$$

Hence,

$$CA_t + KA_t = 0$$

which is the fundamental identity of balance of payments accounting.

A, say, deficit in current account must be necessarily financed by a capital account surplus (*i.e.* borrowing from the rest of the world).

Equilibrium Conditions (cont.)

Alternatively, we can express the current account balance as being equal to saving, S:

$$S_t \equiv rb_t + y_t - c_t \qquad (24)$$

Then

$$CA_t = S_t \qquad (25)$$

Finally, notice that using the definition of the trade balance, given by (19), we can rewrite the intertemporal constraint (18) as

$$\underbrace{\int_0^\infty TB_t e^{-rt} dt}_{\text{NPDV of TB}} = \underbrace{-b_0}_{\text{Initial Net Foreign Debt}}$$

3 Solution of the Model

3.1 General Solution

From $u'(c_t) = \lambda$ (12) it follows that $c_t = \bar{c}$ for all t.

Using this we can use the intertemporal budget constraint (18) to solve for the level of consumption. In particular we have,

$$\int_0^\infty \bar{c} e^{-rt} dt = \bar{c} \int_0^\infty e^{-rt} dt = \bar{c}\left[-\tfrac{1}{r} e^{-rt} \Big|_0^\infty \right] = \bar{c}\left[0 - \left(-\tfrac{1}{r}\right)\right] = \bar{c}\left[\tfrac{1}{r}\right]$$

then

$$\bar{c} = \underbrace{r}_{\text{annuity of}} \underbrace{\left(b_0 + \int_0^\infty y_t e^{-rt} dt \right)}_{\text{permanent income}} \qquad (26)$$

Extension of Friedman (1957) permanent income hypothesis as opposed to Keynes current income $C = f(Y)$.

3.1 General Solution (cont.)

Taking into account $\bar{c} = r\left(b_0 + \int_0^\infty y_t e^{-rt} dt\right)$ (26), the trade balance (19) is given by

$$TB_t = y_t - \bar{c}$$

while the current account is given by

$$CA_t = rb_t + y_t - \bar{c}$$

In good times (high y) the economy will run trade surpluses and the current account will improve, theoretically the opposite should also be true in bad times. In this sense access to international capital markets allows the economy to absorb shocks when faced with a fluctuating path of output.

3.2 Stationary Equilibrium

Def^{n} : a ***stationary equilibrium*** is the perfect foresight equilibrium path corresponding to a flat path of the exogenous variables.

In this case, suppose that the endowment path is given by

$$y_t = y^H \quad \text{for all } t \geq 0 \qquad (27)$$

Rewrite (26) taking into account (27) to get

$$\bar{c} = rb_0 + y^H \qquad (28)$$

Substituting (28) into (19) and taking into account (27) yields:

$$TB_t = -rb_0 \qquad (29)$$

Therefore, in a stationary equilibrium the sign of TB_t dependes on b_0.

Stationary Equilibrium (cont.)

We now show that, in a stationary equilibrium, the current account will always be zero. Substituting $y_t = y^H$ (27) and $\bar{c} = rb_0 + y^H$ (28) into $\dot{b}_t = rb_t + y_t - c_t$ (3), we get

$$\dot{b}_t = rb_t - rb_0$$

This is a standard first-order differential in b_t, whose solution is given by:

$$\dot{b}_t = 0 \qquad \Rightarrow \qquad b_t = b_0 \quad \forall t$$

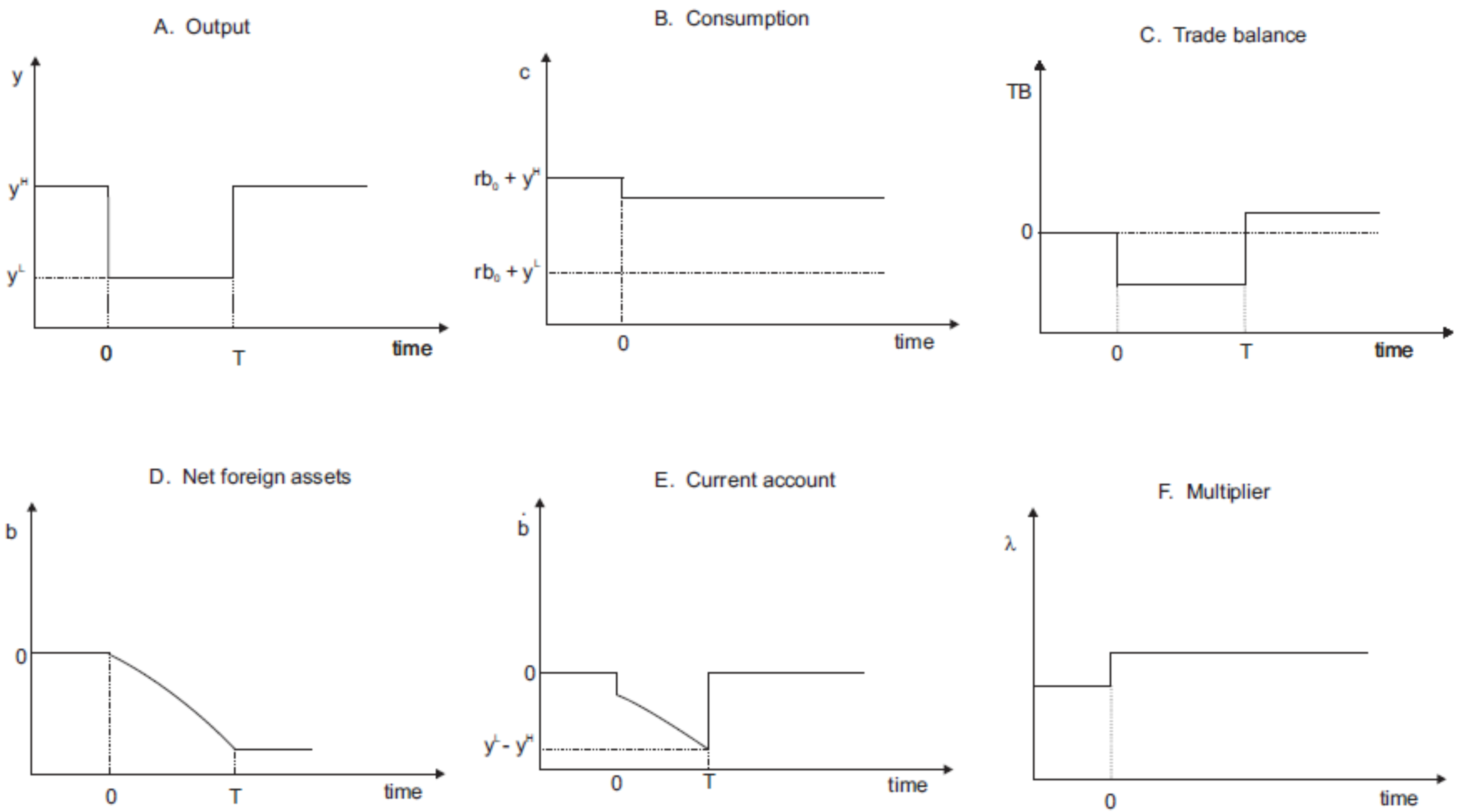
Since net foreign assets are constant over time, the current account is always zero along a stationary equilibrium.

Finally, from $u'(c_t) = \lambda$ (12) and $\bar{c} = rb_0 + y^H$ (28) it follows that

$$\lambda = u'\left(rb_0 + y^H\right)$$

$\therefore$ the value of λ is thus determined by the level of permanent income.

Stationary Equilibrium (this diagrams correspond to temporary shock)



4. Unanticipated Shocks

4.1 Permanent fall in output

From stationary equilibrium of section 3.2, assume that at $t = 0$, there is an unanticipated and permanent fall in output from y^H to y^L, $y^L < y^H$.

Since this was unanticipated, the individual will reoptimize immediately. The F.O.N.C. will now be given by

$$u'(c_t) = \tilde{\lambda}$$

while

$$\bar{c} = rb_0 + y^L$$

4.1 Permanent Fall in Output (cont.)

Since consumption falls one to one with output, the trade balance and the current account do not change.

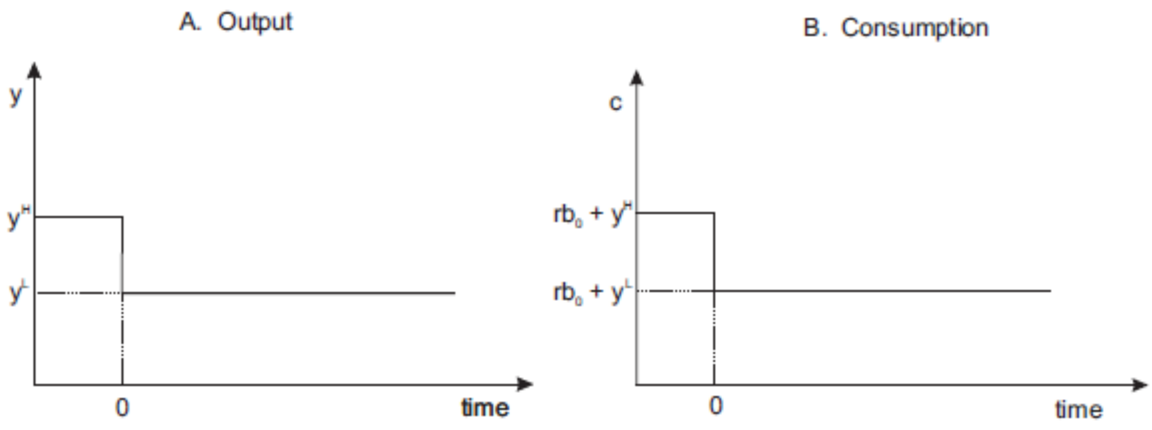
Finally, notice that $\tilde{\lambda} > \lambda$ as

$$\tilde{\lambda} = u'\left(rb_0 + y^L\right)$$

$\therefore$ The economy adjust immediately to a permanent negative shock.

Sincethere are no distortions in this economy, this response is socially optimal.

Permanent Fall in Output (cont.)



4.2 Temporary Fall in Output

From a stationary equilibrium at $t = 0$, we have an unanticipated and temporary fall in output

$$y_t = \begin{cases} y^L & 0 \leq t < T \\ y^H & \text{t} \geq T \end{cases}$$

The intertemporal constraint is now given by :

$$b_0 + \int_0^T y^L e^{-rt} dt + \int_T^\infty y^H e^{-rt} dt = \int_0^\infty c_t e^{-rt} dt \tag{30}$$

The consumer thus maximizes (1) subject to (30).

Consumption F.O.N.C. still implies $c_t = \bar{c} \quad \forall\, t$ with

$$\bar{c} = rb_0 + y^L\left(1 - e^{-rT}\right) + y^H e^{-rT} \tag{31}$$

Temporary Fall in Output (cont.)

Consumption thus falls by the same amount as permanent income, which equals

$$y^H - \left[y^L - y^L e^{-rT} + y^H e^{-rT}\right] = \left(y^H - y^L\right)\left(1 - e^{-rT}\right)$$

Using (31), we can derive the path of trade balance :

$$TB_t = \begin{cases} y^L - \bar{c} = y^L - \left[rb_0 + y^L\left(1 - e^{-rT}\right) + y^H e^{-rT}\right] & \\ \quad = -rb_0 - \left(y^H - y^L\right)e^{-rT} < -rb_0 & 0 \le t < T \\ & \\ y^H - \bar{c} = y^H - \left[rb_0 + y^L\left(1 - e^{-rT}\right) + y^H e^{-rT}\right] & \\ \quad = -rb_0 + \left(y^H - y^L\right)\left(1 - e^{-rT}\right) > -rb_0 & t \ge T \end{cases}$$

Temporary Fall in Output (cont.)

The path of net foreign assets is given by

$$b_t = \begin{cases} b_0 - e^{-r(T-t)}\left(1 - e^{-rT}\right)\frac{\left(y^H - y^L\right)}{r} & 0 \leq t < T \\ b_0 - \left(1 - e^{-rT}\right)\frac{\left(y^H - y^L\right)}{r} & t \geq T \end{cases} \quad (32)$$

A couple of points woth noting : First, b_t is continuous at both $t = 0$ and $t = T$ (but not differentiable). Second, the level of b_t for $t \geq T$ is constant since the economy is, once again, in a stationary equilibrium.

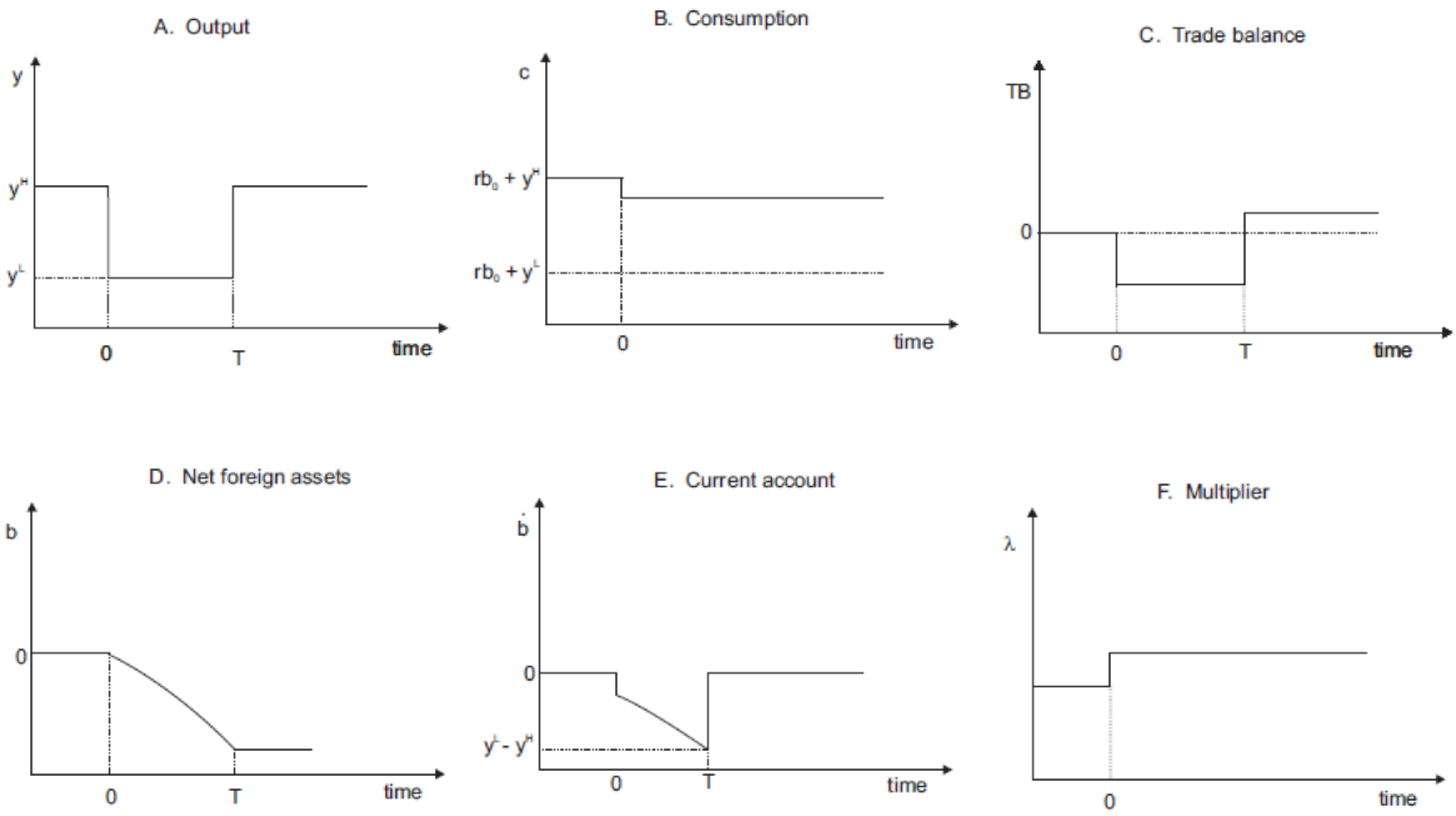
Temporary Fall in Output (cont.)

To derive the path of the current account, we simply differentiate equation (32) $\frac{db_t}{dt}$:

$$\dot{b}_t = \begin{cases} -e^{-r(T-t)}\left(y^H - y^L\right) < 0 & 0 \le t < T \\ 0 & t \ge T \end{cases}$$

Key message: the economy "adjusts" to permanent shocks but "finances" temporary shocks. Since there are no distortions and the economy is thus always operating in a first-best equilibrium, such responses are socially optimal.

Temporary Fall in Output (cont.)



How does our model fit the data? Emerging Markets Business Cycles

	ρ(c,y)	ρ(y,I)	ρ(NX/y,y)	σ(c)/σ(y)
Argentina	0.9	0.96	-0.7	1.38
Brazil	0.41	0.62	0.01	2.01
Ecuador	0.73	0.89	-0.79	2.39
Israel	0.45	0.49	0.12	1.60
Korea	0.85	0.78	-0.61	1.23
Malaysia	0.76	0.86	-0.74	1.70
Mexico	0.92	0.91	-0.74	1.24
Peru	0.78	0.85	-0.24	0.92
Philippines	0.59	0.76	-0.41	0.62
Slovak Republic	0.42	0.46	-0.44	2.04
South Africa	0.72	0.75	-0.54	1.61
Thailand	0.92	0.91	-0.83	1.09
Turkey	0.89	0.83	-0.69	1.09
Mean	**0.72**	**0.77**	**-0.51**	**1.45**

How does our model fit the data? Small Industrial Countries Business Cycles

	ρ(c,y)	ρ(y,I)	ρ(NX/y,y)	σ(c)/σ(y)
Australia	0.48	0.80	-0.43	0.69
Austria	0.74	0.75	0.10	0.87
Belgium	0.67	0.62	-0.04	0.81
Canada	0.88	0.77	-0.20	0.77
Denmark	0.36	0.51	-0.08	1.19
Finland	0.84	0.88	-0.45	0.94
Netherlands	0.72	0.70	-0.19	1.07
New Zealand	0.76	0.82	-0.26	0.90
Norway	0.63	0.00	0.11	1.32
Portugal	0.75	0.70	-0.11	1.02
Spain	0.83	0.83	-0.60	1.11
Sweden	0.35	0.68	0.01	0.97
Switzerland	0.58	0.69	-0.03	0.51
Mean	**0.66**	**0.67**	**-0.17**	**0.94**

How does our model fit the data?

- Data:
  - Strong positive correlation of consumption and output.
  - Strong positive correlation of investment and output.
  - Strong negative correlation of trade balance and output in emerging countries.
  - Consumption more volatile than output in emerging countries and less volatile in industrial countries.

- Model:
  - Perfect correlation of consumption and output in response to unexpected permanent shocks, but neglible correlation in response to unexpected transitory shocks. Furthermore, absent shocks and perfect foresight consumption should be independent of contemporaneous output (?)
  - Not investment yet.
  - The model predicts a positive correlation of trade balance and output (?)
  - The model predicts that consumption volatility should be much lower than output volatility (?)

5. Adding Investment to the Basic Model

- So as we just saw, one problem with the previous model is that it predicts a procyclical trade balance (improves in good times and worsens in bad times) which is contrary to evidence.

- A missing ingredient might be investment.
  - Intuition: a temporary fall in productivity should lead to both lower saving (based on consumption smoothing as before) and lower investment (since productivity is temporarily lower).
  - Therefore the effect would depend on the relative strength of both effects.
  - In particular, if the investment effect dominates, there may be an improvement in the current account.

Adding Investment (cont.)

- Here we show that the relative strength of both effects depends on the duration of the shock.

- Switch from continuous time to discrete time to avoid undefined solution unless investment adjustment cost are introduced. Consider a natural one-period lag in the adjustment of capital.

- Olivier Blanchard (1983) analyzes the continuous-time case with investment adjustment costs in "Debt and the Current Account Deficit in Brazil."

5.1 Household's Problem

5.1.1 Technology

Let y_t denote output that is produced using capital, k_t, according to

$$y_t = A_t f(k_t) \tag{33}$$

where $A_t > 0$ is a productivity parameter and the technology function $f(\cdot)$ satisfies: $f' > 0$ (34) and $f'' < 0$ (35).

Assume no depreciation, then investment I_t follows

$$I_t \equiv k_{t+1} - k_t \tag{36}$$

5.1.2 Household's Budget Constraint

The household's budget constraint at each time t is given by :

$$b_{t+1} = (1+r)b_t + y_t - c_t - (k_{t+1} - k_t) \qquad (37) \sim (3)$$

By iterating forward and imposing the condition

$$\lim_{t \to \infty} \frac{b_t}{(1+r)^{t-1}} = 0$$

We can derive the lifetime budget constraint given by

$$(1+r)b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[c_t + k_{t+1} - k_t\right] \qquad (38) \sim (18)$$

5.1.3 Household's Utility Maximization

$$\max_{\{c_t, k_{t+1}, \lambda\}_{t=0}^{\infty}} \mathrm{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \lambda \left[(1+r) b_0 + \sum_{t=0}^{\infty} \left(\tfrac{1}{1+r}\right)^t A_t f(k_t) - \sum_{t=0}^{\infty} \left(\tfrac{1}{1+r}\right)^t [c_t + k_{t+1} - k_t] \right]$$

The F.O.N.C. for optimization include:

$$\left.\begin{aligned} \tfrac{\partial L}{\partial c_t} &= \beta^t u'(c_t) - \left(\tfrac{1}{1+r}\right)^t \lambda = 0 \\ \tfrac{\partial L}{\partial c_{t+1}} &= \beta^{t+1} u'(c_{t+1}) - \left(\tfrac{1}{1+r}\right)^{t+1} \lambda = 0 \end{aligned}\right\} \qquad u'(c_t) = \beta(1+r) u'(c_{t+1})$$

$$\tfrac{\partial L}{\partial k_{t+1}} = -\left(\tfrac{1}{1+r}\right)^t \lambda + \lambda \left(\tfrac{1}{1+r}\right)^{t+1} [A_{t+1} f'(k_{t+1}) + 1] = 0 \Big\} \qquad r = A_{t+1} f'(k_{t+1}) \qquad (40)$$

$$\tfrac{\partial L}{\partial \lambda} = (1+r) b_0 + \sum_{t=0}^{\infty} \left(\tfrac{1}{1+r}\right)^t A_t f(k_t) - \sum_{t=0}^{\infty} \left(\tfrac{1}{1+r}\right)^t [c_t + k_{t+1} - k_t] = 0$$

If we assume $\beta = \left(\tfrac{1}{1+r}\right)$ the Euler equation implies $\quad u'(c_t) = \lambda \ \ \forall t \qquad (41)$

5.2 Equilibrium Conditions

As before, the change in net foreign assets defines the current account

$$CA_t \equiv b_{t+1} - b_t = rb_t + y_t - c_t - I_t \qquad (42\ \&\ 43 = 44)$$

which is also given by the income balance IB_t and the trade balance TB_t

$$CA_t = \underbrace{IB_t}_{rb_t} + \underbrace{TB_t}_{y_t - c_t - I_t} \qquad (45\ \&\ 46 = 47)$$

or using the determination of savings S_t the current account is also given by

$$CA_t = \underbrace{S_t}_{rb_t + y_t - c_t} - I_t \qquad (48\ \&\ 49)$$

5.3 Perfect Foresight Equilibrium in a Stationary Economy

Assume $A_t = \bar{A} \ \forall \ t = 0,1,\dots$

we can use the FONC for k_{t+1} (40) to get $\bar{k}$

$$\bar{A}f'\left(\bar{k}\right) = r \tag{50}$$

If we assume $k_0 = \bar{k}$, i.e. the economy starts at steady state then $\bar{I} = 0$.

Meanwhile, the constant output is given by

$$\bar{y} = \bar{A}f\left(\bar{k}\right) \tag{51}$$

Perfect Foresight Equilibrium in a Stationary Economy (cont.)

From F.O.N.C. for $c_t : u'(c_t) = \lambda \Rightarrow c_t = \bar{c}$

Furthermore, given that $\bar{I} = 0$ then

$$\bar{c} = rb_0 + \bar{A} f(\bar{k}) \tag{52}$$

Recall $S_t = rb_t + y_t - c_t$. In this case

$$S_0 = rb_0 + y_0 - (rb_0 + y_0) = 0$$

What about S_t for $t = 1, 2, \ldots$?

Perfect Foresight Equilibrium in a Stationary Economy (cont.)

Let's look at CA_t to get b_{t+1}. First $TB_t = y_t - c_t - I_t$

$$\overline{TB} = y_0 - \left(rb_0 + y_0\right) - \underbrace{I_0}_{=0} = -rb_0$$

while

$$CA_0 = S_0 - I_0 = 0$$

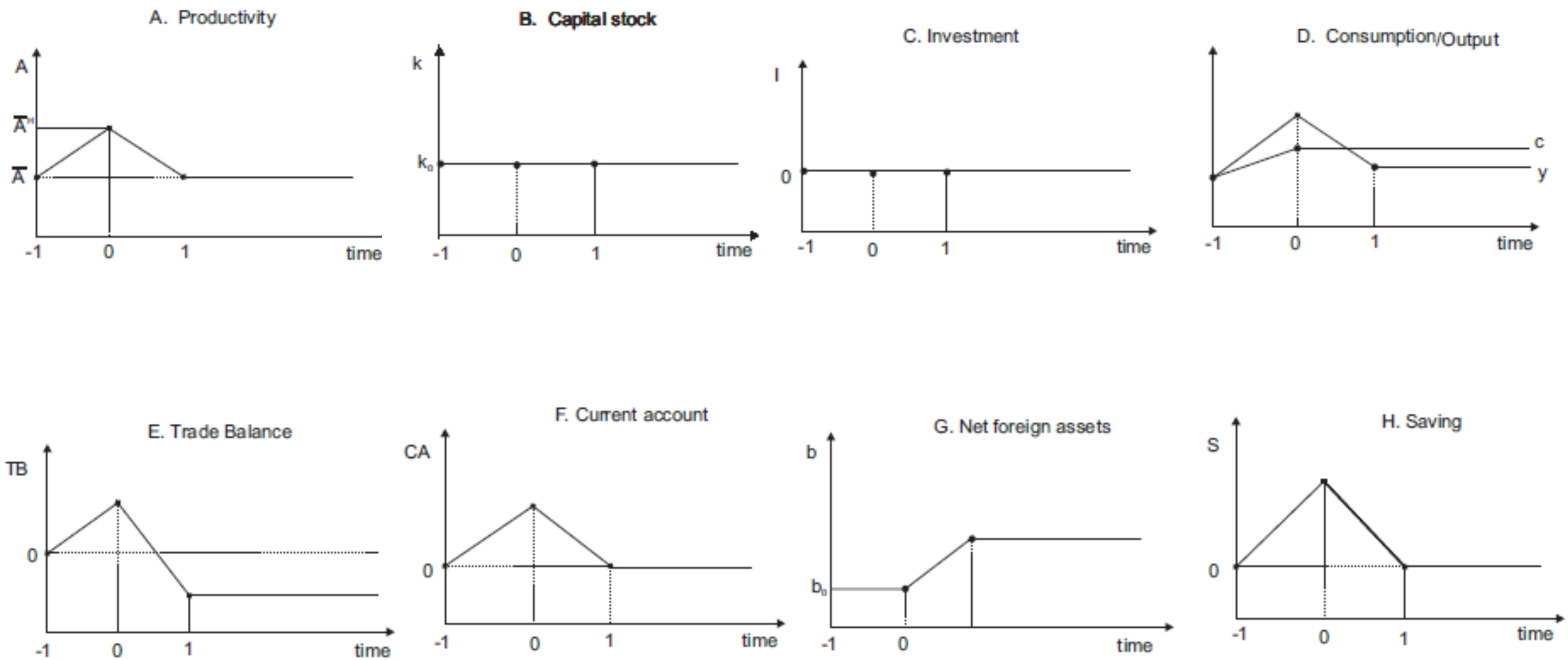
as $CA_t \equiv b_{t+1} - b_t$ and with $CA_0 = 0$ it follows $b_1 = b_0$. Then

$$S_1 = rb_1 + y_1 - \left(rb_0 + y_1\right) = r\left(b_1 - b_0\right) = 0$$

Proceeding in this way with $S_1 = I_1 = 0 \Rightarrow CA_1 = 0 \Rightarrow b_2 = b_1$.

Then $b_{t+1} = b_t \;\forall\; t = 0,1,\ldots$

Stationary Equilibrium of Model with Investment



❖ Note that these diagrams corresponds to one period increase in productivity. The non-distorted equilibrium corresponds to the flat lines everywhere.

5.4 Perfect Foresight Equilibrium for a Growing Economy

Still assume $A_t = \overline{A} \; \forall \; t = 0,1,\ldots$ then $\overline{A}f'(\overline{k}) = r$ (53) still holds, but now assume $k_0 < \overline{k}$

Given (53) $\overline{A}f'(k_0) > r$ then we would like to $\uparrow k$, given the lack of investment adjustment costs we adjust fully such that $k_1 = \overline{k}$. Then

$$I_0 = \overline{k} - k_0 > 0 \qquad (54)$$

$$I_t = 0 \qquad t = 1,2,\ldots$$

while

$$y_0 = \overline{A}f(k_0) \qquad (55)$$

$$y_t = \overline{A}f(\overline{k}) > y_0 \quad t = 1,2,\ldots \qquad (56)$$

Perfect Foresight Equilibrium for a Growing Economy (cont.)

Given that $c_t = \bar{c}$ and the NPV of wealth given y_0, y_t, I_0 and I_t is

$$\bar{c} = rb_0 + \tfrac{r}{1+r}\left[\underbrace{\bar{A}f\left(k_0\right)}_{y_0} + \underbrace{\tfrac{\bar{A}f\left(\bar{k}\right)}{r}}_{\frac{y_t}{r}} - \left(\underbrace{\bar{k} - k_0}_{I_0}\right) - \underbrace{\tfrac{0}{r}}_{\frac{I_t}{r}}\right] \qquad (57)$$

To grasp the intuition behind this expression, rewrite it as :

$$\bar{c} = rb_0 + \bar{A}f\left(k_0\right) + \tfrac{r}{1+r}\left[\underbrace{\frac{\bar{A}\left[f\left(\bar{k}\right) - f\left(k_0\right)\right]}{r} - \left(\bar{k} - k_0\right)}_{\text{NPV of Investment}}\right] \qquad (58)$$

Next step is to show that this NPV of Investment > 0. Rewrite it as

$$\underbrace{\frac{\bar{A}\left[f\left(\bar{k}\right) - f\left(k_0\right)\right]}{\bar{k} - k_0}}_{\text{average return}} > \underbrace{r}_{\substack{\text{marginal} \\ \text{cost}}} = \underbrace{\bar{A}f'\left(\bar{k}\right)}_{\text{marginal return}}$$

which holds given $f'' < 0$ as for $k < \bar{k}$ we have $MPK > r$.

Perfect Foresight Equilibrium for a Growing Economy (cont.)

Since output is low at $t = 0$ relative to $t > 0$, households dissave at $t = 0$ to smooth consumption

$$
\begin{aligned}
S_0 &\equiv rb_0 + \bar{A}f(k_0) - \bar{c}_0 \\
&= rb_0 + \bar{A}f(k_0) - \left[rb_0 + \bar{A}f(k_0) + \tfrac{r}{1+r}\left[\tfrac{\bar{A}[f(\bar{k}) - f(k_0)]}{r} - (\bar{k} - k_0)\right]\right] \\
&= \tfrac{-r}{1+r}\left[\underbrace{\tfrac{\bar{A}[f(\bar{k}) - f(k_0)]}{r} - (\bar{k} - k_0)}_{+}\right] < 0 \qquad (59)
\end{aligned}
$$

$$
\begin{aligned}
TB_0 &= y_0 - c_0 - (\bar{k} - k_0) \\
&= -rb_0 - \underbrace{\tfrac{r}{1+r}\left[\tfrac{\bar{A}[f(\bar{k}) - f(k_0)]}{r} - (\bar{k} - k_0)\right]}_{\text{consumption smoothing effect}} - \underbrace{(\bar{k} - k_0)}_{\text{investment effect}} < -rb_0
\end{aligned}
$$

Perfect Foresight Equilibrium for a Growing Economy (cont.)

To fix ideas assume $b_0 = 0$, then $TB_0 < 0$. If $b_0 = 0$ the NPV of the path of TB must add up to zero. Hence, $TB_t > 0$ for $t = 1,2,\ldots$ with

$$TB_t = -rb_0 + \tfrac{r}{1+r}\left[\frac{\bar{A}\left[f\left(\bar{k}\right) - f\left(k_0\right)\right]}{r} + \left(\bar{k} - k_0\right)\right] > -rb_0$$

which is positive for $b_0 = 0$.

What about CA_t?

$$CA_0 = \underset{(-)}{S_0} - \underset{(+)}{I_0} < 0 \qquad (60)$$

$\therefore$ a "growing" economy should run a current account deficit reflecting negative savings (in anticipation of future output) and positive investment.

Perfect Foresight Equilibrium for a Growing Economy (cont.)

Since in period 1 the economy becomes stationary, the current account should be zero from period 1 onwards. To check this substitute I_0 and S_0 into $CA_0 = S_0 - I_0$ (60) to get

$$CA_0 = -\tfrac{1}{1+r}\left[\overline{A}\left[f\left(\overline{k}\right) - f\left(k_0\right)\right] + \left(\overline{k} - k_0\right)\right] < 0$$

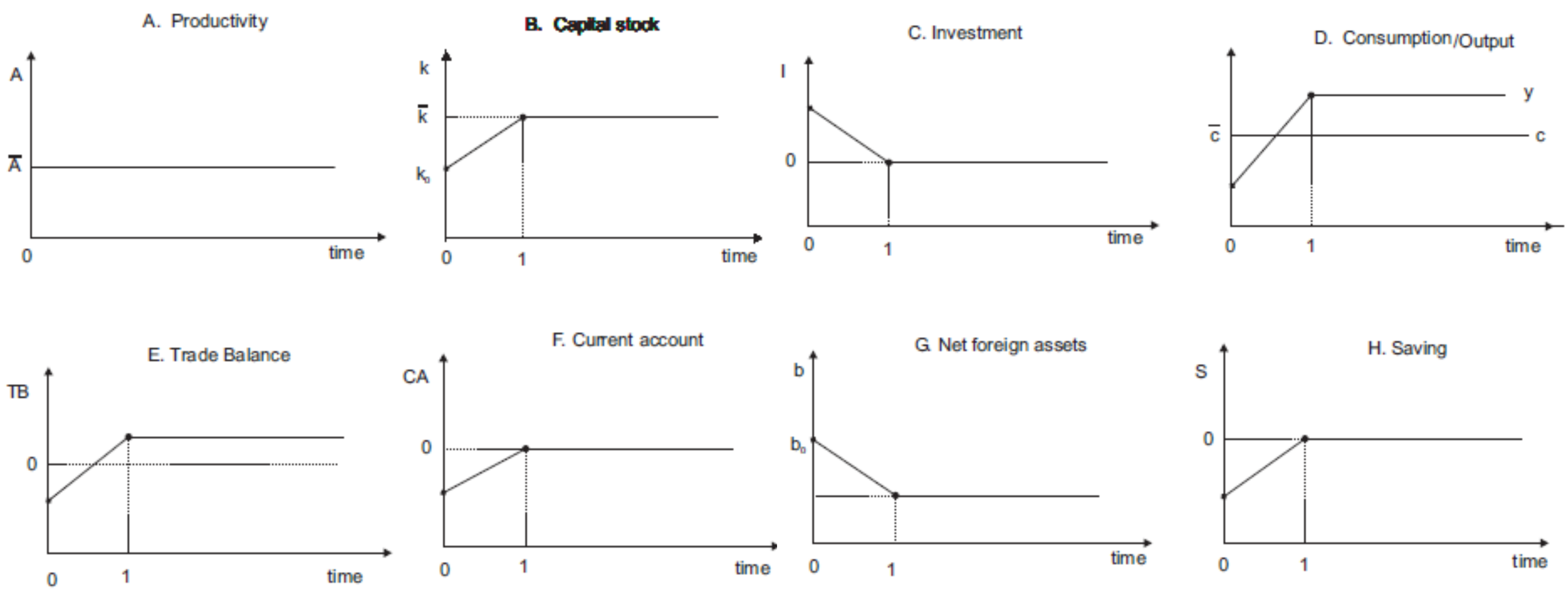
Since $b_{t+1} = b_t + CA_t$, then

$$b_1 = b_0 + \left\{-\tfrac{1}{1+r}\left[\overline{A}\left[f\left(\overline{k}\right) - f\left(k_0\right)\right] + \left(\overline{k} - k_0\right)\right]\right\} < b_0 \qquad (61)$$

Net foreign assets thus decline as a result of the current account deficit in period 0. Then $S_1 = rb_1 + \overline{A}f\left(\overline{k}\right) - c_1$. Using (58) and (61) it is easy to verify $S_1 = 0$. Since $I_1 = 0$ then $CA_1 = 0 \Rightarrow$ (60) to get $b_2 = b_1$. Then

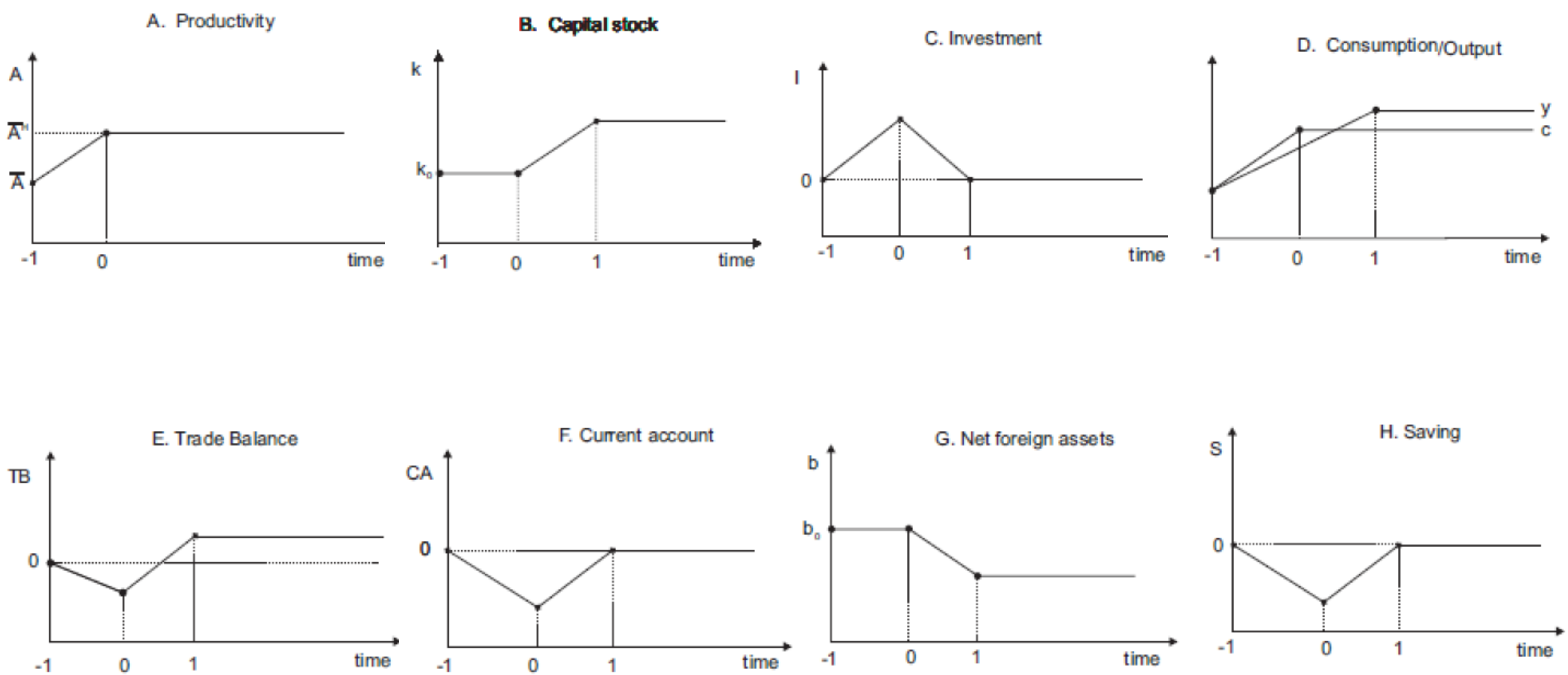
$$S_t = 0 \quad \forall \quad t = 1,2,\ldots \qquad \text{and} \qquad CA_t = 0 \quad \forall \quad t = 1,2,\ldots$$

Perfect Foresight Equilibrium for a Growing Economy (cont.)



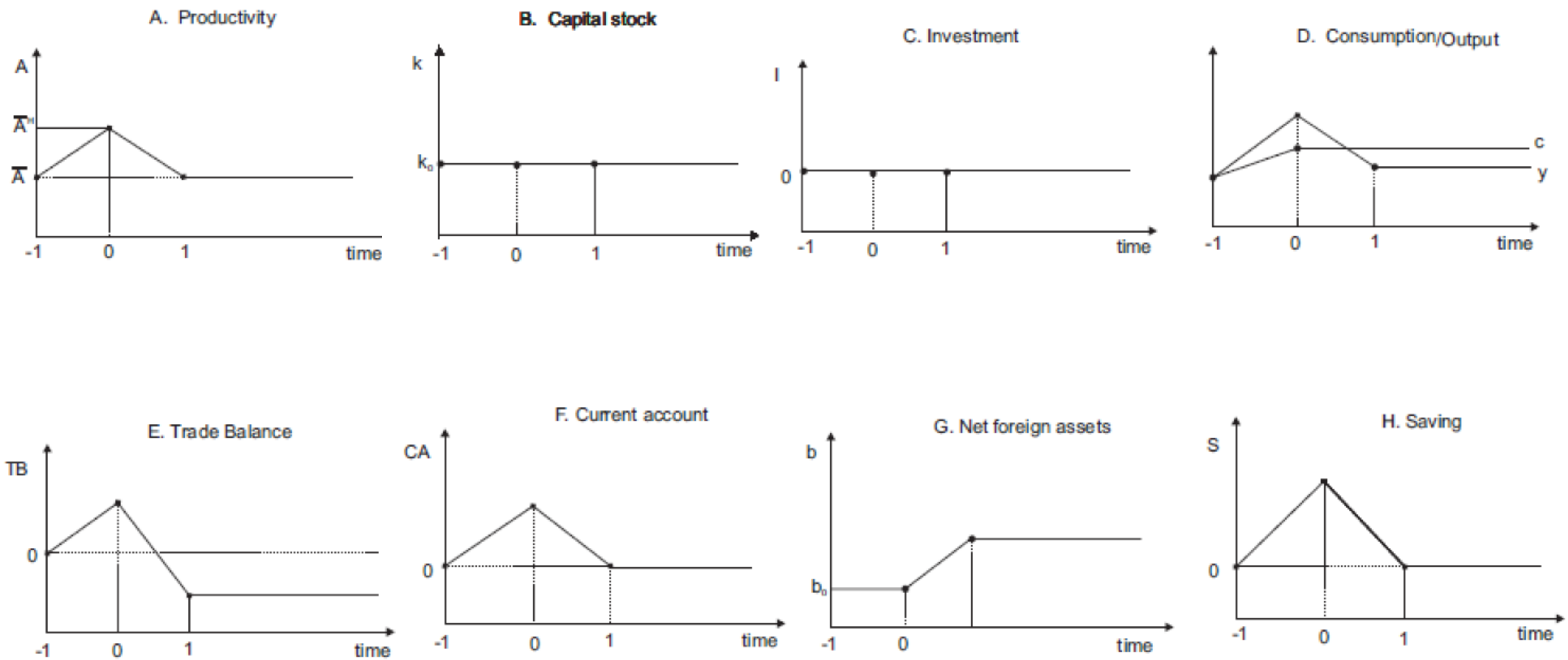
❖ Capital adjusts in one period, the investment is financed with a trade deficit, which reduces net foreign assets and deteriorates the current account. Once the economy grows the economy runs trade surpluses to cover the initial deficit.

5.5 Unanticipated and Permanent Increase in Productivity



❖ In the presence of investment, a permanent increase in productivity leads on impact to trade balance and current account deficits.

5.6 Unanticipated One-Period Increase in Productivity



❖ In sum, an unanticipated one-period rise in productivity leads to a trade balance and current account surpluses. Since *ΔI= 0* this is analogous to the case without investment and a temporary increase in output.

5.7 Unanticipated and Temporary Increase in Productivity

So far we have analyzed two extreme cases:

In section 5.5: Permanent $\uparrow A \quad \rightarrow \quad CA < 0$ as $\downarrow S$ *and* $\uparrow I$

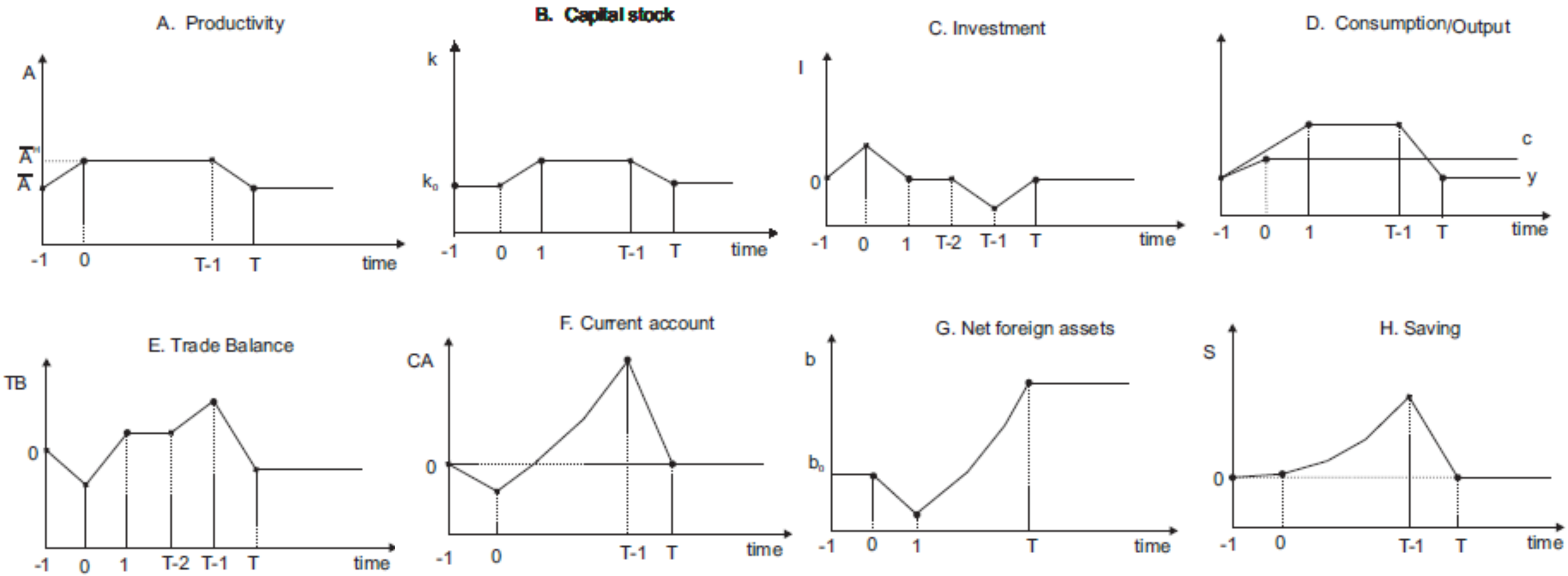
In section 5.6: One period $\uparrow A \quad \rightarrow \quad CA > 0$ as $\uparrow S$ *and* $= I$

$$\Rightarrow \text{an} \uparrow A \text{ may lead to either } CA \gtrless 0$$

For shocks of more than one period, we would conjecture that there will be both a $\uparrow S$ (and eventually $\downarrow S$ as $T \rightarrow \infty$) and $\uparrow I$ and the relative stenght of these two effects will depend on the duration of the shock.

The longer the duration, the smaller the S_0 and hence the more likely $CA_0 < 0$.

Unanticipated and Temporary Increase in Productivity



❖ For shocks of more than one period, we would conjecture that there will be both an increase in savings (that eventually reverses as T becomes larger) and an increase in investment. The relative strength of these two effects will depend on the duration of the shocks. The longer is the duration, the smaller the S_0 and hence the more likely $CA_0<0$.

6. Final Remarks

- This chapter has shown how, in a world with no frictions and a constant world real interest rate, a small open economy will achieve perfect consumption smoothing even when output fluctuates over time.
  - As a benchmark, this is perhaps the most important result of modern open economy macroeconomics.
- In practice, however, developing countries in particular face a variety of frictions that will imply substantial (and welfare reducing) departures from this first-best world.
  - Next week (chapter 2), we will analyze how imperfections in international capital markets may critically affect the economy's ability to achieve consumption smoothing over time.
  - After this, in chapter 3, we will see how policy induced intertemporal distortions lead the private sector to choose non-flat (and socially sub-optimal) consumption paths.