Fundamentals of bond pricing Dr. Alberto Ortiz https://albertoortiz.net/ September 29, 2022

Today's bond price, P_0 , is given by the present discounted value of its expected cash flows, where we discount using the annual yield rate denoted by i. In the case of a bond with an annual coupon rate, cr, on the bond's face value, FV, the semestral cash flows are given by $\frac{cr}{2}*FV$, except at maturity where the cash flow also includes the face value.

One year bond's cash flows are given by:

$$P_0 = \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^1} + \frac{\frac{cr}{2} * FV + FV}{\left(1 + \frac{i}{2}\right)^2}$$

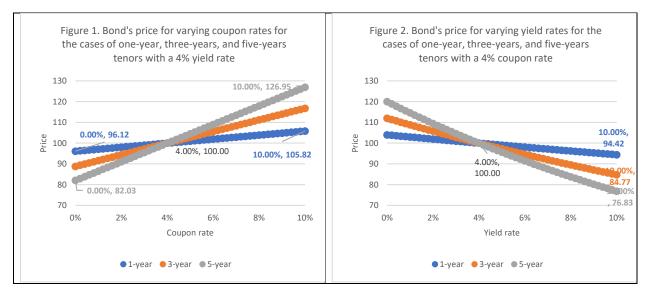
Three-year bond's cash flows are given by:

$$P_{0} = \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^{1}} + \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^{2}} + \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^{3}} + \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^{4}} + \frac{\frac{cr}{2} * FV}{\left(1 + \frac{i}{2}\right)^{5}} + \frac{\frac{cr}{2} * FV + FV}{\left(1 + \frac{i}{2}\right)^{6}}$$

Five-year bond's cash flows are given by:

$$P_0 = \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^1} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^2} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^3} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^4} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^5} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^6} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^7} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^8} + \frac{\frac{cr}{2}*FV}{\left(1+\frac{i}{2}\right)^9} + \frac{\frac{cr}{2}*FV+FV}{\left(1+\frac{i}{2}\right)^{10}}$$

As can be seen from the formulas, the bond's price is increasing in the coupon rate and the face value, and it is decreasing in the yield rate. Figure 1 keeps the yield rate fixed at 4% and shows how the bond's price increases as the coupon rate increases. Figure 2 keeps the coupon rate fixed at 4% and shows how the bond's price decreases as the yield rate increases. Both figures show the relationships for one-, three-, and five-years tenors with a \$100 face value.



Financing cost is given by the yield rate, therefore all the points along each line in Figure 1 have the same financing cost. For example, for a one-year bond depicted by the blue line, yield rate will be the same at 4% if the (coupon rate, price) combinations are (0%, \$96.12), (4%, \$100), and (10%, \$105.82). Meanwhile, for a five-years bond depicted by the gray line, yield rate will be 4% if the (coupon rate, price) combinations are (0%, \$82.03), (4%, \$100), and (10%, \$105.82). Note that if the coupon rate (say 10%) is above the yield rate (say 4%), the longer the tenor (say 5-years), the higher the price of the bond. Meanwhile, if the coupon rate (say 0%) is below the yield rate (say 4%), the longer the tenor, the lower the price of the bond as the opportunity cost of holding it increases. Figure 2 shows the inverse relationship between yield rate and bond's price, where we see that as yield increases, the price of the bond decreases, with bigger drops for longer tenors, as it happens when central bank's raises benchmark interest rates.

Table 1 shows the coupon rates and prices combinations that give a fixed yield rate of 4% for different tenors and are the basis for Figure 1. Table 2 shows the yield rates and prices combinations for a pre-set coupon rate of 4% for different tenors and are the basis for Figure 2.

Table 1: coupon rates and prices combinations for a fixed yield rate of 4%					Table 2: yield rates and prices combinations for a fixed coupon rate of 4%				
tenor	1-year 3-year 5-year			7 _	tenor	1-year	3-year	5-year	
yield	4.00%	4.00%	4.00%		coupon rate	4.00%	4.00%	4.00%	
coupon rate	price	price	price		yield	price	price	price	
10.00%	105.82	116.80	126.95		10.00%	94.42	84.77	76.83	
9.80%	105.63	116.24	126.05		9.80%	94.60	85.23	77.50	
9.60%	105.44	115.68	125.15		9.60%	94.78	85.70	78.17	
9.40%	105.24	115.12	124.25		9.40%	94.96	86.16	78.84	
9.20%	105.05	114.56	123.35		9.20%	95.14	86.63	79.53	
9.00%	104.85	114.00	122.46		9.00%	95.32	87.11	80.22	
8.80%	104.66	113.44	121.56		8.80%	95.50	87.58	80.92	
8.60%	104.47	112.88	120.66		8.60%	95.68	88.06	81.62	
8.40%	104.27	112.32	119.76		8.40%	95.86	88.54	82.33	
8.20%	104.08	111.76	118.86		8.20%	96.04	89.03	83.05	
8.00%	103.88	111.20	117.97		8.00%	96.23	89.52	83.78	
7.80%	103.69	110.64	117.07		7.80%	96.41	90.01	84.51	
7.60%	103.49	110.08	116.17		7.60%	96.60	90.50	85.25	
7.40%	103.30	109.52	115.27		7.40%	96.78	91.00	86.00	
7.20%	103.11	108.96	114.37		7.20%	96.96	91.50	86.76	
7.00%	102.91	108.40	113.47		7.00%	97.15	92.01	87.53	
6.80%	102.72	107.84	112.58		6.80%	97.34	92.52	88.30	
6.60%	102.52	107.28	111.68		6.60%	97.52	93.03	89.08	
6.40%	102.33	106.72	110.78		6.40%	97.71	93.54	89.87	
6.20%	102.14	106.16	109.88		6.20%	97.90	94.06	90.66	
6.00%	101.94	105.60	108.98		6.00%	98.09	94.58	91.47	
5.80%	101.75	105.04	108.08		5.80%	98.28	95.11	92.28	
5.60%	101.55	104.48	107.19		5.60%	98.46	95.64	93.11	
5.40%	101.36	103.92	106.29		5.40%	98.65	96.17	93.94	
5.20%	101.16	103.36	105.39		5.20%	98.85	96.71	94.78	
5.00%	100.97	102.80	104.49		5.00%	99.04	97.25	95.62	
4.80%	100.78	102.24	103.59		4.80%	99.23	97.79	96.48	
4.60%	100.58	101.68	102.69		4.60%	99.42	98.34	97.35	
4.40%	100.39	101.12	101.80		4.40%	99.61	98.89	98.22	
4.20%	100.19	100.56	100.90		4.20%	99.81	99.44	99.11	
4.00%	100.00	100.00	100.00		4.00%	100.00	100.00	100.00	
3.80%	99.81	99.44	99.10		3.80%	100.19	100.56	100.9	
3.60%	99.61	98.88	98.20		3.60%	100.39	101.13	101.83	
3.40%	99.42	98.32	97.31		3.40%	100.59	101.70	102.74	
3.20%	99.22	97.76	96.41	∐Ħ	3.20%	100.78	102.27	103.6	
3.00%	99.03	97.20	95.51		3.00%	100.98	102.85	104.63	
2.80%	98.84	96.64	94.61		2.80%	101.18	103.43	105.56	
2.60%	98.64	96.08	93.71		2.60%	101.37	104.02	106.52	
2.40%	98.45	95.52	92.81		2.40%	101.57	104.60	107.50	
2.20%	98.25	94.96	91.92	∐Ħ	2.20%	101.77	105.20	108.48	
2.00%	98.06	94.40	91.02		2.00%	101.97	105.80	109.47	
1.80%	97.86	93.84	90.12		1.80%	102.17	106.40	110.47	
1.60%	97.67	93.28	89.22		1.60%	102.37	107.00	111.49	
1.40%	97.48	92.72	88.32		1.40%	102.57	107.61	112.53	
1.20%	97.28	92.16	87.42		1.20%	102.78	108.23	113.5	
1.00%	97.09	91.60	86.53		1.00%	102.98	108.84	114.60	
0.80%	96.89	91.04	85.63		0.80%	103.18	109.47	115.65	
0.60%	96.70	90.48	84.73		0.60%	103.38	110.09	116.72	
0.40%	96.51	89.92	83.83		0.40%	103.59	110.72	117.80	
0.20%	96.31	89.36	82.93		0.20%	103.79	111.36	118.90	
0.00%	96.12	88.80	82.03		0.00%	104.00	112.00	120.00	