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Estimating contract indexation in a Financial Accelerator Model ☆

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ABSTRACT

This paper addresses the positive implications of indexing risky debt to observable aggregate conditions. These issues are pursued within the context of the celebrated financial accelerator model of Bernanke et al. (1999). The principal conclusions include: (1) the estimated level of indexation is significant, (2) the business cycle properties of the model are significantly affected by this degree of indexation, (3) the importance of investment shocks in the business cycle depends upon the estimated level of indexation, and (4) although the data prefers the financial model with indexation over the frictionless model, they have remarkably similar business cycle properties for *non-financial* exogenous shocks.

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1. Introduction

The fundamental function of credit markets is to channel funds from savers to entrepreneurs who have some valuable capital investment project. These efforts are hindered by agency costs arising from asymmetric information. A standard result in a subset of this literature, the costly state verification (CSV) framework, is that risky debt is the optimal contract between risk-neutral lenders and entrepreneurs. The modifier risky simply means that there is a non-zero chance of default. In the CSV model external parties can observe the realization of the entrepreneur's idiosyncratic production technology only by expending a monitoring cost. Townsend (1979) demonstrates that risky debt is optimal in this environment because it

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minimizes the need for verification of project outcomes. This verification is costly but necessary to align the incentives of the firm with the bank.

Aggregate conditions will also affect the ability of the borrower to repay the loan. But since aggregate variables are observed by both parties, it may be advantageous to have the loan contract indexed to the behavior of aggregate variables. Therefore, even when loan contracts cannot be designed based on private information, we can exploit common information to make these financial contracts more state-contingent. That is, why should the loan contract call for costly monitoring when the event that leads to a poor return is observable by all parties? Carlstrom et al. (2013),[1] hereafter CFP, examine questions of this type within the financial accelerator of Bernanke et al. (1999), hereafter BGG. CFP demonstrate that the privately optimal contract in the BGG model includes indexation to: (i) the aggregate return to capital (which we will call R^k-indexation), (ii) the household's marginal utility of wealth, and, (iii) the entrepreneur's valuation of internal funds. CFP also demonstrate that the latter two forms of indexation quantitatively largely cancel out, so our focus will be on R^k-indexation.

In this paper we thus explore the business cycle implications of indexing the BGG loan contract to the aggregate return to capital. There are at least two reasons why this is an interesting exercise. First, as noted above, CFP demonstrate that the privately optimal contract in the BGG framework includes indexation of this very type. Second, indexation of this type is not so far removed from some financial contracts we do observe. Since we are assuming that the CSV framework proxies for agency cost effects in the entire US financial system, it seems reasonable to include some form of indexation to mimic the myriad ex post returns on external financing. For example, in contrast to the model assumption where entrepreneurs get zero in the event of bankruptcy, this is clearly not the implication of Chapter 11 bankruptcy. In any event, we use familiar Bayesian methods to estimate the degree of contract indexation to the return to capital.

To avoid misspecification problems in the estimation we need a complete model of the business cycle. We use the recent contribution of Justiniano et al. (2011), hereafter JPT, as our benchmark. A novelty of the JPT model is that it includes two shocks to the capital accumulation technology. The first shock is a non-stationary shock to the relative cost of producing investment goods, the "investment specific technology shock" (IST). The second is a stationary shock to the transformation of investment goods into installed capital, the "marginal efficiency of investment shock" (MEI). For business cycle variability, JPT find that the IST shocks are irrelevant, while the MEI shocks account for a substantial portion of business cycle fluctuations.

Our principal results include the following. First, the estimated level of R^k-indexation significantly exceeds unity, much higher than the assumed BGG indexation of approximately zero. A model with R^k-indexation fits the data significantly better when compared to BGG. This is because the BGG model's prediction for the risk premium in the wake of a MEI shock is counterfactual. A MEI shock lowers the price of capital and thus leads to a sharp decline in entrepreneurial net worth in the BGG model. But under R^k-indexation, the required repayment falls also so that net worth moves by significantly less.

Second, with R^k-indexation, this financial model and JPT have remarkably similar business cycle properties for *non-financial* exogenous shocks. For example, for the case of MEI shocks, the estimated level of indexation leads to net worth movements in the financial model that accommodate real behavior quite similar to the response of JPT to an MEI shock. We also nest financial shocks into the JPT model by treating fluctuations in the financial variables as serially correlated measurement error. This model horserace results in the R^k-indexation model dominating BGG, which in turn significantly dominates JPT. The financial models are improvements over JPT in two ways. The financial models make predictions for the risk premium on which JPT is silent, and the financial models introduce other exogenous shocks, e.g., shocks to net worth or idiosyncratic variance, that are irrelevant in JPT.

Third, we find that whether financial shocks or MEI shocks are more important drivers of the business cycle depends upon the level of indexation. Under BGG, financial shocks account for a significant part of the variance of investment spending. But under the estimated level of R^k-indexation, financial shocks become much less important and the MEI shocks are again of paramount importance.

Two prominent papers closely related to the current work are Christiano et al. (2014), and DeGraeve (2008). They each use Bayesian methods to estimate versions of the BGG framework in medium-scale macro models. Both papers conclude that the model with financial frictions provides a better fit to the data when compared to its frictionless counterpart. The chief novelty of the current paper is to introduce contract indexation into the BGG framework, and demonstrate that it is empirically relevant, altering the business cycle properties of the model. Neither of the previous papers considered indexation of this type.

The paper proceeds as follows. Section 2 presents a simple example that illustrates the importance of contract indexation to the financial accelerator. Section 3 develops the DSGE model. Section 4 presents the estimation results, while Section 5 provides some sensitivity analysis. Section 6 concludes.

2. Why does indexation matter? A simple example

This section presents a simple intuitive example that demonstrates the importance of indexation in determining the size of the financial accelerator. Consider a world with agency costs in which the portion of net worth owned by entrepreneurs

[1] This is the logic behind Shiller and Weiss's (1999) suggestion of indexing home mortgages to movements in aggregate house prices.

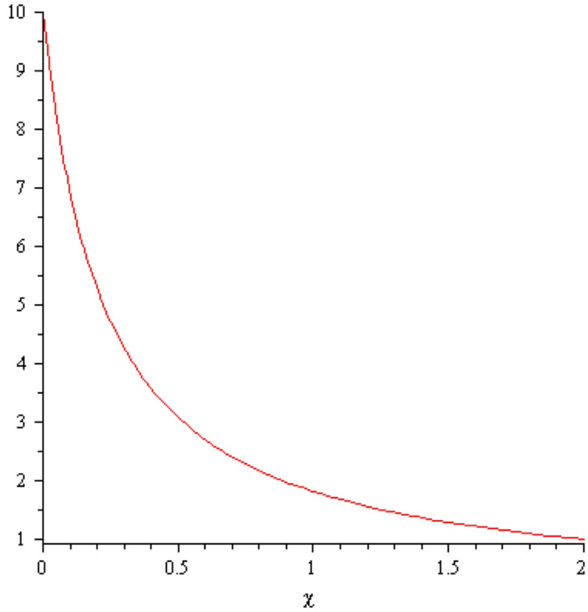

Fig. 1. The multiplier as a function of indexation.

(nw_t) has a positive effect on the value of capital (q_t):

$$q_t = pnw_t + \epsilon_t^d \tag{1}$$

where the expression is in log deviations and ϵ_t^d is an exogenous shock to capital prices, e.g., a shock to MEI in the general equilibrium model below. Eq. (1) is a manifestation of agency costs in that the distribution of net worth across lenders and borrowers affects asset prices. The idea is that higher net worth in the hands of entrepreneurs makes it easier for them to access a loan with which to buy capital, so that higher levels of net worth act like a demand channel on asset prices. In the general equilibrium model below, the value of p is a function of the agency cost and (installed) capital adjustment cost parameters.

The entrepreneur accumulates net worth to mitigate the agency problems involved in direct lending. The agency problem arises from a CSV problem in the entrepreneur's production technology. The entrepreneur takes one unit of input and creates ω_t units of capital, where the unit-mean random variable ω_t is privately observed by the entrepreneur but can be verified by the lender only by paying a cost. This CSV problem makes equity finance problematic, so that the optimal contract is given by a risky debt contract with a promised repayment of r_t^p. The repayment r_t^p cannot be indexed to ω_t because it is privately observed. But it can be indexed to the aggregate price of capital:

$$r_t^p = \chi q_t. \tag{2}$$

This form of indexation is similar to indexing to the rate of return on capital in the general equilibrium model developed below.

Entrepreneurial net worth accumulates with the profit flow from the investment project, but decays via consumption of entrepreneurs (which is a constant fraction of net worth). Log-linearized this evolution is given by:

$$nw_t = \kappa(q_t - r_t^p) + nw_{t-1} + r_t^p + \epsilon_t^n \tag{3}$$

where $\kappa > 1$ denotes leverage (the ratio of project size to net worth) and ϵ_t^n is an exogenous shock to net worth. Using the indexation assumption (2), we can express (3) as

$$nw_t = q_t[\kappa - \chi(\kappa - 1)] + nw_{t-1} + \epsilon_t^n \tag{4}$$

Note that since $\kappa > 1$, the slope of the net worth equation is decreasing in the level of indexation.

Eqs. (1) and (4) are a simultaneous system in net worth and the price of capital. We can solve for the two endogenous variables as a function of the pre-determined and exogenous variables:

$$nw_t = \frac{nw_{t-1} + \epsilon_t^n + \epsilon_t^d[\kappa - \chi(\kappa - 1)]}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \tag{5}$$

$$q_t = \frac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \tag{6}$$

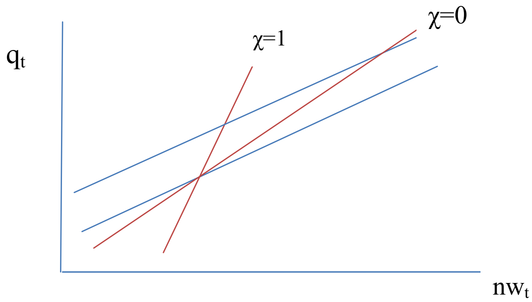

Fig. 2. A shock to asset demand. (Asset price is blue line. Net worth evolution is red line.) Demand shock shifts up asset price. The new equilibrium in (n,q) space depends upon the level of indexation. Lower levels of indexation amplify these effects. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The inverse of the denominator in (5) and (6) is the familiar "multiplier" arising from two endogenous variables with positive feedback. This then implies that exogenous shocks are "multiplied" or "financially accelerated", and that the degree of this multiplication depends upon the level of indexation. The effect of indexation on the financial multiplier is highly nonlinear. Fig. 1 plots the multiplier for $\kappa=2$, and $p=0.45$, both of these values roughly correspond to the general equilibrium analysis below. Note that moving from $\chi=0$ to $\chi=1$, has an enormous effect on the multiplier. But there are sharp diminishing returns so the multiplier is little changed as we move from $\chi=1$ to $\chi=2$. This suggests, and we confirm below, that the data can distinguish $\chi=0$ from, say, $\chi=1$, but that indexation values in excess of unity will have similar business cycle characteristics and thus be difficult to identify.

Consider three special cases of indexation: $\chi=0$, $\chi=1$, and $\chi=\kappa/(\kappa-1)$ The first is the implicit assumption in BGG; the second implies complete indexation; the third eliminates the financial accelerator altogether. In these cases, net worth and asset prices are given by:

Indexation	Net worth	Capital price	Multiplier ($p=0.45$, $\kappa=2$)
$\chi=0$	$\frac{nw_{t-1}+\epsilon_t^n+\kappa\epsilon_t^d}{1-p\kappa}$	$\frac{p(nw_{t-1}+\epsilon_t^n)+\epsilon_t^d}{1-p\kappa}$	10
$\chi=1$	$\frac{nw_{t-1}+\epsilon_t^n+\epsilon_t^d}{1-p}$	$\frac{p(nw_{t-1}+\epsilon_t^n)+\epsilon_t^d}{1-p}$	1.82
$\chi=\kappa/(\kappa-1)$	$nw_{t-1}+\epsilon_t^n$	$p(nw_{t-1}+\epsilon_t^n)+\epsilon_t^d$	1

For both $\chi=0$ and $\chi=1$, exogenous shocks to asset prices and net worth have multiple effects on the equilibrium levels of net worth and capital prices. Since $\kappa>1$, this effect is much larger under BGG's assumption of no indexation $(1/(1-p\kappa)\gg 1/(1-p))$. Further, under the BGG assumption, exogenous shocks to asset prices (ϵ_t^d) have an added effect as they are weighted by leverage. But for all levels of indexation, there are always agency cost effects in that the price of capital is affected by the level of entrepreneurial net worth. The financial multiplier effects are traced out in Fig. 2: an exogenous shock to asset prices has a much larger effect on both net worth and asset prices in the BGG framework. Finally, since $\kappa\approx 2$ the financial accelerator largely disappears when $\chi=2$.

Before proceeding, it is helpful to emphasize the two parameters that are crucial in our simple example as they will be manifested below in the richer general equilibrium environment. Our reduced form parameter p in Eq. (1) is the agency cost parameter. In a Modigliani-Miller world we would have $p=0$, as the distribution of net worth would have no effect on asset prices or real activity. Second, the indexation parameter χ determines the size of the financial accelerator, i.e., how do unexpected movements in asset prices feed in to net worth? These are two related but logically distinct ideas. That is, one can imagine a world with agency costs ($p>0$), but with very modest accelerator effects ($\chi=\kappa/(\kappa-1)$.). To anticipate our empirical results, this is the parameter set that wins the model horse race. That is, the data is consistent with an agency cost model but with trivial accelerator effects. In such an environment, financial shocks (e.g., shocks to net worth) will affect real activity, but other real shocks (e.g., MEI shocks) will not be accelerated.

3. The model

The benchmark model follows the JPT framework closely. The model of agency costs comes from BGG with the addition of exogenous contract indexation. The BGG loan contract is between lenders and entrepreneurs, so we focus on these two agents first before turning to the familiar framework of JPT.

3.1. Lenders

The representative lender accepts deposits from households (promising a sure real return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time

$t+1$. The realized gross real return on these loans is denoted by R^L_{t+1}. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only the aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, real dividends are given by $Div_{t+1} = (R^L_{t+1}D_t - R^d_t D_t)$. The intermediary seeks to maximize its equity value which is given by:

$$Q^L_t = E_t \sum_{j=1}^{\infty} \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} \mathrm{Div}_{t+j} \tag{7}$$

where Λ_t is the marginal utility of real income for the representative household that owns the lender.

The FOC of the lender's problem is:

$$E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R^L_{t+1} - R^d_t\right] = 0 \tag{8}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur.[2] We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, e.g., Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (e.g., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.[3]

3.2. Entrepreneurs and the loan contract

There are a continuum of entrepreneurs, but as in BGG there is sufficient linearity in preferences and technology such that we can abstract to a representative entrepreneur. Entrepreneurs are the sole accumulators of physical capital. At the beginning of period t, the entrepreneurs sell all of their accumulated capital to "capital agencies" at beginning-of-period capital price Q^{beg}_t. At the end of the period, the entrepreneurs purchase the entire capital stock $\overline{K}_t$, including any net additions to the stock, at end-of-period price Q_t. This re-purchase of capital is financed with entrepreneurial net worth (NW_t) and external financing from a lender. Loan size or credit is thus defined as $Credit_t \equiv Q_t \overline{K}_t - NW_t$. This external finance takes the form of a one period loan contract. The gross return to holding capital from time-t to time $t+1$ is given by:

$$R^k_{t+1} \equiv \frac{Q^{beg}_{t+1}}{Q_t}. \tag{9}$$

Below we show that $Q^{beg}_{t+1} = Q_{t+1}(1-\delta) + [\rho_{t+1}u_{t+1} - a(u_{t+1})]$, where ρ_{t+1} and u_{t+1} denote the rental rate and utilization rate of capital, respectively. Variations in R^k_{t+1} are the source of aggregate risk in the loan contract. The external financing is subject to a costly-state-verification (CSV) problem because of idiosyncratic risk. In particular, one unit of capital purchased at time-t is transformed into ω_{t+1} units of capital in time $t+1$, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$. The realization of ω_{t+1} is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid, a cost that is fraction μ_{mc} of the project outcome. Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let R^p_{t+1} denote the promised gross rate-of-return so that R^p_{t+1} is defined by

$$R^p_{t+1}(Q_t\overline{K}_t - NW_t) \equiv \varpi_{t+1} R^k_{t+1} Q_t \overline{K}_t. \tag{10}$$

We find it convenient to express this in terms of the leverage ratio $\overline{\kappa}_t \equiv (Q_t\overline{K}_t / NW_t)$ so that (10) becomes

$$R^p_{t+1} \equiv \varpi_{t+1} R^k_{t+1} \frac{\overline{\kappa}_t}{\overline{\kappa}_t - 1} \tag{11}$$

Let $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denote the entrepreneur's share and lender's share of the project outcome:

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\omega)]\varpi \tag{12}$$

$$g(\varpi) \equiv [1-\Phi(\varpi)]\varpi + (1-\mu_{mc}) \int_0^{\varpi} \omega\phi(\omega)d\omega \tag{13}$$

Using these expressions, the lender's ex post realized $t+1$ return on the loan contract is defined as:

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[2] In contrast, BGG assume that bank profits are always zero ex post so that the lender's return is pre-determined. This is not a feature of the optimal contract. See Carlstrom et al. (2013) for details.

[3] Alternatively we could make the deposit rate state-contingent (and thus equal to the lending rate) so that profits are always zero. This alternative decentralization would have no effect on the model.

$$R_{t+1}^{L} \equiv \frac{R_{t+1}^{k} g(\varpi_{t+1}) Q_t \overline{K}_t}{Q_t \overline{K}_t - NW_t} = R_{t+1}^{k} g(\varpi_{t+1}) \frac{\overline{\kappa}_t}{\overline{\kappa}_t - 1} \tag{14}$$

Similarly, the ex post $t+1$ payoff to the entrepreneur with net worth NW_t is given by:

$$\text{Entrepreneur's payoff} = \overline{\kappa}_t NW_t R_{t+1}^{k} f(\varpi_{t+1}) \tag{15}$$

The optimal loan contract will maximize the entrepreneur's payoff subject to the lender's participation constraint.

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure that there is a need for external finance in the long run, we assume that fraction $(1-\gamma)$ of the entrepreneurs die each period, where death entails eating their accumulated net worth and exiting the economy. These dying entrepreneurs are replaced by an inflow of new entrepreneurs each with a trivial initial level of net worth.

CFP show that the entrepreneur's value function is linear in net worth. Let $V_t NW_t$ denote this value function after loans have been repaid, but before the death shock is realized. Given the trivial consumption-savings decision, the Bellman equation is given by:

$$V_t NW_t = (1-\gamma) NW_t + \max_{\overline{\kappa}_t, \varpi_{t+1}} \gamma \beta E_t V_{t+1} \overline{\kappa}_t NW_t R_{t+1}^{k} f(\varpi_{t+1}) \tag{16}$$

where the maximization is subject to the lender's participation constraint. Recall that the lender's stochastic discount factor comes from the household, and the lender's return is linked to the return on deposits via (8), so that the lender's participation constraint is given by:

$$E_t R_{t+1}^{L} \Lambda_{t+1} = R_t^{d} E_t \Lambda_{t+1} \tag{17}$$

Hence, the end-of-time-t contracting problem can be written as:

$$\max_{\overline{\kappa}_t,\, \varpi_{t+1}} \{ E_t V_{t+1} R_{t+1}^{k} \overline{\kappa}_t f(\varpi_{t+1}) \} \tag{18}$$

subject to

$$E_t R_{t+1}^{k} \frac{\overline{\kappa}_t}{\overline{\kappa}_t - 1} \Lambda_{t+1} g(\varpi_{t+1}) \geq R_t^{d} E_t \Lambda_{t+1} \tag{19}$$

An important observation is that the choice of ϖ_{t+1} can be made contingent on public information available in time $t+1$. Indexation of this type is optimal. After some re-arrangement, the contract optimization conditions include:

$$V_{t+1} f'(\varpi_{t+1}) = \left[\frac{E_t V_{t+1} f'(\varpi_{t+1})}{E_t \Lambda_{t+1} g'(\varpi_{t+1})} \right] \Lambda_{t+1} g'(\varpi_{t+1}) \tag{20}$$

$$(\overline{\kappa}_t - 1) E_t V_{t+1} R_{t+1}^{k} f(\varpi_{t+1}) = \left[\frac{-E_t V_{t+1} f'(\varpi_{t+1})}{E_t \Lambda_{t+1} g'(\varpi_{t+1})} \right] E_t \Lambda_{t+1} R_{t+1}^{k} g(\varpi_{t+1}) \tag{21}$$

$$E_t \Lambda_{t+1} R_{t+1}^{k} \frac{\overline{\kappa}_t}{\overline{\kappa}_t - 1} g(\varpi_{t+1}) = R_t^{d} E_t \Lambda_{t+1} \tag{22}$$

A key result is given by (20). The optimal monitoring cut-off ϖ_{t+1} is independent of innovations in R_{t+1}^{k}. The second-order condition implies that the ratio $(-f'(\varpi_{t+1})/(g'(\varpi_{t+1})))$, is increasing in ϖ_{t+1}. Hence, another implication of the privately optimal contract is that ϖ_{t+1} (and thus the optimal repayment rate R_{t+1}^{p}) is an increasing function of (innovations in) the marginal utility of wealth Λ_{t+1}, and a decreasing function of (innovations in) the entrepreneur's valuation V_{t+1}.

The monitoring cut-off implies the behavior of the repayment rate (see (11)). In log deviations, the repayment rate under the optimal contract is given by:

$$\hat{r}_t^{p} = \hat{r}_{t-1}^{d} + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} \hat{\kappa}_{t-1} + (\hat{r}_t^{k} - E_{t-1}\hat{r}_t^{k}) + \frac{1}{\Psi}(\hat{\lambda}_t - E_{t-1}\hat{\lambda}_t) - \frac{1}{\Psi}(\hat{v}_t - E_{t-1}\hat{v}_t) \tag{23}$$

$$\hat{v}_t = E_t \sum_{j=0}^{\infty} \beta^{j+1} (\Xi \hat{\kappa}_{t+j} + \hat{r}_{t+j+1}^{k}) \tag{24}$$

where the hatted lower case letters denote log deviations of their uppercase counterparts, and the positive constants Θ_g, ν, Ξ, and Ψ are defined in the appendix. The analysis of CFP demonstrates that the latter two terms in (23) largely cancel out quantitatively. That is, shocks that lead to positive innovations in the marginal utility of wealth also lead to symmetric innovations in the return to internal funds. Hence, for parsimony we will estimate an indexation rule of the form:

$$\hat{r}_t^{p} = \hat{r}_{t-1}^{d} + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} \hat{\kappa}_{t-1} + \chi_k(\hat{r}_t^{k} - E_{t-1}\hat{r}_t^{k}) \tag{25}$$

As noted in the example sketched in Section 2, different indexation values will have dramatic effects on the financial accelerator. The original BGG model assumes that the lender's return was pre-determined. From (14), this implies that χ_k is small but modestly negative ($\chi_k = -0.05$, in our benchmark calibration).

Summing across all entrepreneurs, aggregate net worth accumulation is described by

$$NW_t = \gamma NW_{t-1}\overline{\kappa}_{t-1}R_t^k f(\varpi_t)\eta_{nw,t} \tag{26}$$

where $\eta_{nw,t}$ is an exogenous disturbance to the distribution of net worth.[4] This disturbance follows the stochastic process

$$\log \eta_{nw,t} = \rho_{nw}\log \eta_{nw,t-1} + \varepsilon_{nw,t}, \tag{27}$$

with $\varepsilon_{nw,t}$ *i.i.d.* $N(0,\sigma_{nw}^2)$. Eq. (26) implies that NW_t is determined by the realization of R_t^k and the response of ϖ_t to these realizations. NW_t then enters the contracting problem in time t so that the realization of R_t^k is propagated forward.

As in Christiano et al. (2010, 2014), and Gilchrist et al. (2009), we also consider time variation in the variance of the idiosyncratic shock ω_t. The variance of ω_t is denoted by σ_t and follows the exogenous stochastic process given by

$$\log \sigma_t = \rho_\sigma \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \tag{28}$$

Shocks to this variance will alter the risk premium in the model.

3.3. Final good producers

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\lambda_{p,t})}di\right]^{1+\lambda_{p,t}} \tag{29}$$

The elasticity $\lambda_{p,t}$ follows the exogenous stochastic process

$$\log \lambda_{p,t} = (1-\rho_p)\log \lambda_p + \rho_p \log \lambda_{p,t-1} + \varepsilon_{p,t} - \theta_p\varepsilon_{p,t-1}, \tag{30}$$

where $\varepsilon_{p,t}$ is *i.i.d.* $N(0,\sigma_p^2)$. Fluctuations in this elasticity are price markup shocks. Profit maximization and the zero profit condition imply that the price of the final good, P_t, is the familiar CES aggregate of the prices of the intermediate goods.

3.4. Intermediate goods producers

A monopolist produces the intermediate good i according to the production function

$$Y_t(i) = \max\{A_t^{1-\alpha}K_t(i)^\alpha L_t(i)^{1-\alpha} - A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}F; 0\}, \tag{31}$$

where $K_t(i)$ and $L_t(i)$ denote the amounts of capital and labor employed by firm i. F is a fixed cost of production, chosen so that profits are zero in steady state. The variable A_t is the exogenous non-stationary level of TFP progress. Its growth rate ($z_t \equiv \Delta \ln A_t$) is given by

$$z_t = (1-\rho_z)\gamma_z + \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{32}$$

with $\varepsilon_{z,t}$ *i.i.d.* $N(0,\sigma_z^2)$. The other non-stationary process Υ_t is linked to the investment sector and is discussed below.

Every period a fraction ξ_p of intermediate firms cannot choose its price optimally, but resets it according to the indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}, \tag{33}$$

where $\pi_t \equiv P_t/P_{t-1}$ is gross inflation and π is its steady state. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^{\infty}\xi_p^s\frac{\beta^s\Lambda_{t+s}/P_{t+s}}{\Lambda_t/P_t}\left[P_t(i)\left(\prod_{k=1}^{s}\pi_{t+k-1}^{\iota_p}\pi^{1-\iota_p}\right)Y_{t+s}(i) - W_{t+s}L_{t+s}(i) - P_{t+s}\rho_{t+s}K_{t+s}(i)\right]\right\} \tag{34}$$

where the demand function comes from the final goods producers, Λ_t/P_t is the marginal utility of nominal income for the representative household, and W_t is the nominal wage.

3.5. Employment agencies

Firms are owned by a continuum of households, indexed by $j \in [0,1]$. Each household is a monopolistic supplier of specialized labor, $L_t(j)$, as in Erceg et al. (2000). A large number of competitive employment agencies combine this specialized labor into a homogenous labor input sold to intermediate firms, according to

$$L_t = \left[\int_0^1 L_t(j)^{1/(1+\lambda_{w,t})}dj\right]^{1+\lambda_{w,t}} \tag{35}$$

[4] This disturbance to net worth is assumed to only alter the net worth of the new entrepreneurs so that it does not enter the Bellman equation above.

As in the case of the final good, the desired markup of wages over the household's marginal rate of substitution, $\lambda_{w,t}$, follows the exogenous stochastic process

$$\log \lambda_{w,t} = (1-\rho_w)log\lambda_w + \rho_w \log \lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}, \tag{36}$$

with $\varepsilon_{w,t}$ *i.i.d.* N $(0, \sigma_w^2)$. This is the wage markup shock. Profit maximization by the perfectly competitive employment agencies implies that the wage paid by intermediate firms for their homogenous labor input is

$$W_t = \left[\int_0^1 W_t(j)^{-1/\lambda_{w,t}} dj\right]^{-\lambda_{w,t}} \tag{37}$$

3.6. Capital agencies

The capital stock is managed by a collection of perfectly competitive capital agencies. These firms are owned by households and discount cash flows with Λ_t, the marginal utility of real income for the representative household. At the beginning of period t, these agencies purchase the capital stock $\overline{K}_{t-1}$ from the entrepreneurs at beginning-of-period price Q_t^{beg}. The agencies produce capital services by varying the utilization rate u_t which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}. \tag{38}$$

Effective capital is then rented to firms at the real rental rate ρ_t.The cost of capital utilization is $a(u_t)$ per unit of physical capital. The capital agency then re-sells the capital to entrepreneurs at the end of the period at price Q_t. The profit flow is thus given by:

$$Q_t(1-\delta)\overline{K}_{t-1} + [\rho_t u_t - a(u_t)]\overline{K}_{t-1} - Q_t^{beg}\overline{K}_{t-1} \tag{39}$$

Profit maximization implies

$$Q_t^{beg} = Q_t(1-\delta) + [\rho_t u_t - a(u_t)] \tag{40}$$

$$\rho_t = a'(u_t) \tag{41}$$

In steady state, $u=1$, $a(1)=0$ and $\vartheta \equiv a''(1)/a'(1)$. Hence, in the neighborhood of the steady state

$$Q_t^{beg} \approx Q_t(1-\delta) + \rho_t \tag{42}$$

which is consistent with BGG's definition of the intertemporal return to holding capital $R_t^k \equiv ((\rho_t + (1-\delta)Q_t)/(Q_{t-1}))$.

These capital agencies also augment the capital stock by producing investment goods. Investment goods are produced with a linear technology that transforms one consumption good into Υ_t investment goods. The exogenous level of productivity Υ_t is non-stationary with a growth rate ($\upsilon_t \equiv \Delta \log Y_t$) given by

$$\upsilon_t = (1-\rho_\upsilon)\gamma_\upsilon + \rho_\upsilon \upsilon_{t-1} + \varepsilon_{\upsilon,t}. \tag{43}$$

The constant returns production function implies that the price of investment goods (in consumption units) is equal to $P_t^I = (1/\Upsilon_t)$. These investment goods are transformed into new capital via a concave technology that takes I_t investment goods and transforms them into $\mu_t[1-S(I_t/I_{t-1})]I_t$ new capital goods. The time-t profit flow is thus given by

$$Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t^I I_t \tag{44}$$

where P_t^I is the relative price of the investment good. The function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005). The function has the following convenient steady state properties: $S=S'=0$ *and* $S''>0$. These firms are owned by households and discount future cash flows with Λ_t, the marginal utility of real income for the representative household. JPT refer to the investment shock μ_t as a shock to the marginal efficiency of investment (MEI) as it alters the transformation between investment and installed capital. JPT conclude that this shock is the primary driver of output and investment at business cycle frequencies. The investment shock follows the stochastic process

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \tag{45}$$

where $\varepsilon_{\mu,t}$ is *i.i.d.* N $(0, \sigma_\mu^2)$.

3.7. Households

Each household maximizes the utility function

$$E_t\left\{\sum_{s=0}^{\infty} \beta^s b_{t+s}\left[\ln(C_{t+s} - hC_{t+s-1}) - \varphi \frac{L_{t+s}(j)^{1+\psi}}{1+\psi}\right]\right\}, \tag{46}$$

where C_t is consumption, h is the degree of habit formation and b_t is a shock to the discount factor. This intertemporal preference shock follows the stochastic process

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t}, \tag{47}$$

with $\varepsilon_{b,t} \sim i.i.d.\ N(0, \sigma_b^2)$. Since technological progress is nonstationary, utility is logarithmic to ensure the existence of a balanced growth path. The existence of state contingent securities ensures that household consumption is the same across all households. The household's flow budget constraint is

$$C_t + T_t + D_t + \frac{B_t}{P_t} \le \frac{R_{t-1}B_{t-1}}{P_t} + \frac{W_t(j)}{P_t}L_t(j) + R^d_{t-1}D_{t-1} + profits_t, \tag{48}$$

where D_t denotes real deposit at the lender, T_t is lump-sum taxes, and B_t is holdings of nominal government bonds that pay gross nominal rate R_t. The term $profits_t$ denotes the combined profit flow of all the firms owned by the representative agent including lenders, intermediate goods producers, and capital agencies. Every period a fraction ξ_w of households cannot freely set its wage, but follows the indexation rule

$$W_t(j) = W_{t-1}(j)(\pi_{t-1}e^{z_{t-1}+\frac{\alpha}{1-\alpha}\upsilon_t})^{\iota_w}(\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_\upsilon})^{1-\iota_w}, \tag{49}$$

The remaining fraction of households chooses instead an optimal wage $W_t(j)$ by maximizing

$$E_t\left\{\sum_{s=0}^{\infty}\xi_w^s\beta^s\left[-b_{t+s}\varphi\frac{L_{t+s}(j)^{1+\psi}}{1+\psi} + \frac{\Lambda_{t+s}}{P_{t+s}}W_t(j)L_{t+s}(j)\right]\right\} \tag{50}$$

subject to the labor demand function coming from the firm.

3.8. The government

A monetary policy authority sets the nominal interest rate following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{X_t}{X_t^*}\right)^{\phi_x}\right]^{1-\rho_R}\left[\frac{X_t/X_{t-1}}{X_t^*/X_{t-1}^*}\right]^{\phi_{dx}}\eta_{mp,t}, \tag{51}$$

where R is the steady state of the gross nominal interest rate. The interest rates responds to deviations of inflation from its steady state, as well as to the level and the growth rate of the GDP gap (X_t/X_t^*). The monetary policy rule is also perturbed by a monetary policy shock, $\eta_{mp,t}$, which evolves according to

$$\log \eta_{mp,t} = \rho_{mp} \log \eta_{mp,t-1} + \varepsilon_{mp,t}, \tag{52}$$

where $\varepsilon_{mp,t}$ is $i.i.d.\ N(0, \sigma_{mp}^2)$. Public spending is determined exogenously as a time-varying fraction of output.

$$G_t = \left(1 - \frac{1}{g_t}\right)Y_t, \tag{53}$$

where the government spending shock g_t follows the stochastic process

$$\log g_t = (1-\rho_g)\log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}, \tag{54}$$

with $\varepsilon_{g,t} \sim i.i.d.\ N(0, \sigma_g^2)$. The spending is financed with lump sum taxes.

3.9. Market clearing

The aggregate resource constraints are given by:

$$C_t + \frac{I_t}{\Upsilon_t} + G_t + a(u_t)\overline{K}_{t-1} = Y_t \tag{55}$$

$$\overline{K}_t = (1-\delta)\left(1 - \mu_{mc}\int_0^{\varpi_t}\omega\phi(\omega)d\omega\right)\overline{K}_{t-1} + \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t, \tag{56}$$

In (55), we follow Christiano et al. (2014), and assume that the trivial level of entrepreneurial consumption is transferred to households for their consumption. This completes the description of the model. We now turn to the estimation of the linearized model.

4. Estimation

The linearized version of the model equations are collected in the appendix. The three fundamental agency cost parameters are the steady state idiosyncratic variance (σ_{ss}), the entrepreneurial survival rate (γ), and the monitoring cost fraction (μ_{mc}). In contrast to DeGraeve (2008), we follow Christiano et al. (2010), and calibrate these parameters to be consistent with long run aspects of US financial data. We follow this calibration approach because these parameters are

Table 1
Models estimations and models comparisons with BAA – T10 credit spread and return to capital data. All models have autocorrelated measurement errors in the credit spread and return to capital series.

					JPT Model[a]			BGG Model[b]			Indexation to R^k Model[c]		
Log data density					−1429.1			−1383.1			−1339.6		
Posterior model probability					0%			0%			100%		
Coefficient	**Description**	**Prior**			**Posteriors**[e]			**Posteriors**			**Posteriors**		
		Prior density[d]	**Prior mean**	**pstdev**	**post. mean**	**5%**	**95%**	**Post. mean**	**5%**	**95%**	**Post. mean**	**5%**	**95%**
α	**Capital share**	**N**	0.30	0.05	0.16	0.15	0.17	0.17	0.15	0.18	0.16	0.15	0.17
ι_p	**Price indexation**	**B**	0.50	0.15	0.31	0.12	0.54	0.13	0.05	0.21	0.15	0.07	0.23
ι_w	**Wage indexation**	**B**	0.50	0.15	0.18	0.10	0.26	0.12	0.06	0.19	0.17	0.11	0.23
γ_z	**SS technology growth rate**	**N**	0.50	0.03	0.51	0.47	0.54	0.49	0.48	0.51	0.49	0.46	0.53
γ_υ	**SS IST growth rate**	**N**	0.50	0.03	0.50	0.48	0.52	0.49	0.47	0.51	0.51	0.48	0.54
h	**Consumption habit**	**B**	0.50	0.10	0.87	0.83	0.91	0.85	0.81	0.88	0.88	0.86	0.91
λ_p	**SS mark-up goods prices**	**N**	0.15	0.05	0.18	0.13	0.25	0.20	0.15	0.25	0.18	0.14	0.23
λ_w	**SS mark-up wages**	**N**	0.15	0.05	0.17	0.14	0.20	0.15	0.11	0.20	0.08	0.04	0.12
$\log L^{ss}$	**SS hours**	**N**	0.00	0.50	0.34	−0.18	1.08	0.40	0.02	0.73	0.33	−0.03	0.64
$100(\pi - 1)$	**SS quarterly inflation**	**N**	0.50	0.10	0.64	0.49	0.82	0.63	0.57	0.69	0.72	0.67	0.77
$100(\beta^{-1} - 1)$	**Discount factor**	**G**	0.25	0.10	0.11	0.05	0.17	0.14	0.07	0.22	0.14	0.07	0.22
Ψ	**Inverse frisch elasticity**	**G**	2.00	0.75	2.43	1.62	3.18	4.64	4.28	5.02	3.39	2.53	4.09
ξ_p	**Calvo prices**	**B**	0.66	0.10	0.72	0.67	0.77	0.79	0.73	0.85	0.74	0.70	0.78
ξ_w	**Calvo wages**	**B**	0.66	0.10	0.50	0.40	0.61	0.87	0.81	0.93	0.66	0.60	0.71
ϑ	**Elasticity capital utilization costs**	**G**	5.00	1.00	5.62	4.28	6.89	4.68	4.15	5.12	5.17	4.30	5.91
$\ddot{S}$	**Investment adjustment costs**	**G**	4.00	1.00	2.96	2.55	3.33	1.09	0.83	1.33	2.43	1.90	2.96
ϕ_π	**Taylor rule inflation**	**N**	1.70	0.30	1.94	1.65	2.27	1.35	1.19	1.51	2.11	1.91	2.31
ϕ_x	**Taylor rule gap**	**N**	0.13	0.05	0.06	0.04	0.10	0.04	0.01	0.07	0.11	0.09	0.13
ϕ_{dx}	**Taylor rule gap growth**	**N**	0.13	0.05	0.18	0.13	0.23	0.27	0.24	0.30	0.20	0.16	0.23
ρ_R	**Taylor rule smoothing**	**B**	0.60	0.20	0.77	0.73	0.81	0.83	0.80	0.87	0.86	0.84	0.89
ρ_{mp}	**Monetary policy**	**B**	0.40	0.20	0.05	0.00	0.09	0.06	0.01	0.11	0.05	0.01	0.10
ρ_z	**Neutral technology growth**	**B**	0.60	0.20	0.38	0.29	0.48	0.24	0.13	0.33	0.37	0.30	0.44
ρ_g	**Government spending**	**B**	0.60	0.20	1.00	1.00	1.00	0.99	0.99	1.00	1.00	0.99	1.00
ρ_υ	**IST growth**	**B**	0.60	0.20	0.26	0.17	0.36	0.43	0.31	0.52	0.35	0.26	0.44
ρ_p	**Price mark-up**	**B**	0.60	0.20	0.95	0.91	1.00	0.91	0.83	0.98	0.95	0.91	0.99
ρ_w	**Wage mark-up**	**B**	0.60	0.20	0.99	0.98	1.00	0.95	0.91	0.99	0.96	0.94	0.98
ρ_b	**Intertemporal preference**	**B**	0.60	0.20	0.54	0.44	0.64	0.56	0.44	0.70	0.58	0.50	0.66
θ_p	**Price mark-up MA**	**B**	0.50	0.20	0.61	0.49	0.71	0.75	0.66	0.84	0.71	0.62	0.81
θ_w	**Wage mark-up MA**	**B**	0.50	0.20	0.82	0.70	0.93	0.95	0.89	1.00	0.99	0.98	1.00
ρ_σ	**Idiosyncratic variance**	**B**	0.60	0.20	–	–	–	0.94	0.88	1.00	0.87	0.77	0.96
ρ_{nw}	**Net worth**	**B**	0.60	0.20	–	–	–	0.79	0.73	0.86	0.78	0.71	0.85
ρ_μ	**Marginal efficiency of investment**	**B**	0.60	0.20	0.70	0.65	0.76	0.17	0.04	0.28	0.89	0.77	0.99
ρ_{rpme}	**Risk premium measurement error**	**B**	0.60	0.20	0.75	0.52	0.97	0.95	0.91	0.99	0.72	0.61	0.83
ρ_{rkme}	**R^k measurement error**	**B**	0.60	0.20	0.59	0.48	0.70	0.60	0.48	0.73	0.79	0.65	0.91
ν	**Elasticity risk premium**	**N**	0.05	0.02	0	–	–	0.19	–	–	0.19	–	–
χ_k	**Indexation to R^k**	**U**	0.00	2.00	0	–	–	BGG	–	–	2.24	1.70	2.82

Standard deviation of shocks

		Prior density	Prior mean	Pstdev	Post. mean	5%	95%	Post. mean	5%	95%	Post. mean	5%	95%
σ_{mp}	**Monetary policy**	**I**	0.20	1.00	0.27	0.24	0.30	0.25	0.22	0.27	0.22	0.20	0.25
σ_z	**Neutral technology growth**	**I**	0.50	1.00	0.89	0.80	0.98	0.89	0.80	0.98	0.88	0.79	0.97
σ_g	**Government spending**	**I**	0.50	1.00	0.36	0.33	0.40	0.36	0.33	0.40	0.36	0.33	0.40
σ_υ	**IST growth**	**I**	0.50	1.00	0.58	0.53	0.63	0.59	0.53	0.64	0.58	0.53	0.64
σ_p	**Price mark-up**	**I**	0.10	1.00	0.23	0.19	0.27	0.27	0.23	0.31	0.24	0.21	0.28
σ_w	**Wage mark-up**	**I**	0.10	1.00	0.29	0.24	0.33	0.31	0.25	0.36	0.35	0.31	0.39
σ_b	**Intertemporal preference**	**I**	0.10	1.00	0.04	0.03	0.05	0.03	0.02	0.05	0.03	0.02	0.04
σ_σ	**Idiosyncratic variance**	**I**	0.50	1.00	–	–	–	0.08	0.06	0.08	0.08	0.07	0.09
σ_{nw}	**Net worth**	**I**	0.50	1.00	–	–	–	0.87	0.69	1.07	1.43	1.12	1.74
σ_μ	**Marginal efficiency of investment**	**I**	0.50	1.00	5.59	4.76	6.28	2.39	1.80	3.01	3.67	2.96	4.41

Table 1 (*continued*)

Standard deviation of shocks													
		Prior density	Prior mean	Pstdev	Post. mean	5%	95%	Post. mean	5%	95%	Post. mean	5%	95%
σ_{rpme}	**Risk premium measurement error**	I	0.50	1.00	0.19	0.17	0.20	0.08	0.07	0.09	0.08	0.06	0.08
σ_{rkme}	**R^k measurement error**	I	0.50	1.00	2.35	2.12	2.59	2.67	2.39	2.99	1.52	1.08	2.08

Note: Calibrated coefficients: $\delta=0.025$, g implies a SS government share of 0.22.For the agency cost models (BGG and Indexation) the following parameters are also calibrated: entrepreneurial survival rate $\gamma=0.94$, a SS risk premium $rp=0.02/4$, and a SS leverage ratio $\kappa=1.95$.

[a] In JPT model there are not financial (risk premium and net worth) shocks. The elasticity of risk premium, ν, is set to 0, and the indexation parameter χ_k is irrelevant and set to 0.

[b] In BGG model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.19, while the indexation to the return of capital parameter, χ_k, is set to the implied in BGG, $\chi_k=(\Theta_g-1)/\Theta_g$ where $\Theta_g=0.95$.

[c] In the Indexation to R^k model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.19 and the indexation to the return of capital parameter, χ_k, is estimated.

[d] N stands for Norman, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution.

[e] Posterior percentiles are from 2 chains of 100,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 50,000 and retain one every 5 subsequent draws.

pinned down by long run or steady state properties of the model, not the business cycle dynamics that the Bayesian estimation is trying to match. In any event, these three parameters are calibrated to match the steady state levels of the risk premium (R^p-R^d), leverage ratio ($\overline{\kappa}$), and default rate ($\Phi(\varpi_{ss})$). In particular, they are chosen to deliver a 200 bp annual risk premium (BAA-Treasury spread), a leverage ratio of $\overline{\kappa}=1.95$, and a quarterly default rate of 0.03/4. These imply an entrepreneurial survival rate of $\gamma=0.944$, a standard deviation of $\sigma_{ss}=0.2865$, and a monitoring cost of $\mu_{mc}=0.63$.[5] A key expression in the log-linearized model is the reduced-form relationship between leverage and the investment distortion:

$$E_t\hat{r}^k_{t+1}-\hat{r}^d_t=\nu(\hat{q}_t+\hat{k}_t-\hat{n}_t)+\hat{\sigma}_t \tag{57}$$

(See the Appendix for details.) The value of ν implied by the previous calibration is $\nu=0.19$. This is thus imposed in the estimation of the financial models.[6] For JPT we have $\nu=0$. Steady state relationships also imply that we calibrate $\delta=0.025$, and $(1-1/g)=0.22$. The remaining parameters are estimated using familiar Bayesian techniques as in JPT. For the non-financial parameters of the model we use the same priors as in JPT.

We treat as observables the growth rates of real GDP, consumption, investment, the real wage, and the relative price of investment. The other observables include employment, inflation, the nominal rate, the return on capital (R^k_t), and the risk premium ($E_t\hat{r}^p_{t+1}-\hat{r}^d_t$). Employment is measured as the log of per capita hours. Inflation is the consumption deflator, and the nominal rate is the federal funds rate. The series for R^k_t comes from Gomme et al. (2011). The risk premium is the spread between the BAA and ten year Treasury. The time period for the estimation is 1972:1–2008:4. We choose the end of the sample period to avoid the observed zero bound on the nominal rate.

We estimate three versions of the model. Along with all the exogenous shocks outlined in the paper, we also include autocorrelated measurement error between the model's risk premium and the observed risk premium. Autocorrelated measurement error is also included for R^k_t. The first model we label JPT as it corresponds to the model without agency costs ($\nu=0$). Note that the JPT model will assign all risk premium variation to autocorrelated measurement error. The remaining two models have operative agency costs ($\nu=0.19$). Recall that the optimal contract has the form given by:

$$\hat{r}^p_t=\hat{r}^d_{t-1}+\frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\hat{\kappa}_{t-1}+\chi_k(\hat{r}^k_t-E_{t-1}\hat{r}^k_t) \tag{58}$$

For both models, we impose the values of ν, κ, and Θ_g, as these are pinned down by the steady state of the model. In the model labeled BGG we also impose the level of indexation implicitly assumed by BGG: $\chi_k=-0.05$. For the model labeled R^k-indexation, we estimate the value of χ_k. We use diffuse priors on the indexation parameters with a uniform distribution centered at 0 and with a standard deviation of 2.

The agency cost models also include two financial shocks: (i) time-varying movements in idiosyncratic risk, and (ii) exogenous redistributions of net worth. Both of these shocks are irrelevant in the JPT model in which lending is not subject to the CSV problem. We posit priors for the standard deviation and autocorrelation of these financial shocks in a manner symmetric with the non-financial exogenous processes in JPT.

[5] This monitoring cost is not as large as it seems. In Carlstrom and Fuerst (1997), monitoring costs are given by $\mu_{mc}\int_0^{\varpi_t}\phi(\omega)d\omega=\mu_{mc}\Phi(\varpi_t)$. This is of course larger than the expression in (56). Using this expression for monitoring costs, the calibration in the text would require $\mu_{mc}=0.29$, within the range of reasonable numbers suggested by Carlstrom and Fuerst (1997). It is unclear which expression for monitoring costs is more appropriate.

[6] As a form of sensitivity analysis, we also estimated ν in the financial models. We found that the estimation is quite sensitive to priors, again suggesting that it is not well identified by business cycle dynamics.

Table 2
Variance decomposition at different horizons in the JPT, BGG and R^k Indexation Models.

Output

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	3.4	14.5	4.8	2.2	8.1	6.7	5.7	54.6	-	-	0.0
BGG	16.4	12.9	5.2	0.4	5.7	0.2	3.3	3.5	49.5	2.8	0.0
R^k Indexation	2.2	18.3	5.6	2.2	4.9	5.0	8.1	45.5	7.4	0.8	0.0
8 quarters											
JPT	3.3	7.8	3.3	1.6	12.1	16.9	3.8	51.3	-	-	0.0
BGG	15.2	6.4	3.6	0.2	7.9	2.4	2.9	1.7	57.2	2.5	0.0
R^k Indexation	2.3	10.4	4.4	1.5	8.2	13.5	6.6	41.1	11.1	1.0	0.0
16 quarters											
JPT	2.5	5.3	2.7	1.2	12.3	34.7	2.4	38.9	-	-	0.0
BGG	12.8	6.0	3.3	0.2	8.2	10.0	2.5	1.1	53.8	2.1	0.0
R^k Indexation	1.8	7.2	4.2	1.1	9.4	29.6	4.7	30.4	10.8	1.0	0.0
1000 quarters											
JPT	0.8	1.8	15.0	0.4	4.6	62.9	0.8	13.5	-	-	0.0
BGG	7.6	4.3	22.9	0.2	4.9	24.8	1.5	0.6	32.0	1.2	0.0
R^k Indexation	1.8	7.2	4.2	1.1	9.4	29.6	4.7	30.4	10.8	1.0	0.0

Investment

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	2.6	4.4	0.0	0.3	7.3	1.6	2.1	81.7	-	-	0.0
BGG	10.6	2.7	0.0	0.7	3.7	1.1	3.4	3.6	71.7	2.4	0.0
R^k Indexation	0.8	1.8	0.0	0.3	3.2	0.5	1.0	72.1	19.5	0.7	0.0
8 quarters											
JPT	2.3	10.6	0.0	0.3	10.3	4.2	2.7	69.6	-	-	0.0
BGG	7.8	5.9	0.0	1.5	4.6	0.6	2.2	1.7	74.0	1.7	0.0
R^k Indexation	0.6	6.2	0.0	0.7	5.0	1.5	1.2	57.5	26.4	0.8	0.0
16 quarters											
JPT	1.9	15.6	0.0	0.9	11.8	9.2	2.8	57.7	-	-	0.0
BGG	6.5	9.0	0.0	2.5	4.9	1.2	1.8	1.4	71.2	1.4	0.0
R^k Indexation	0.5	10.5	0.0	2.1	6.1	3.2	1.1	48.4	27.2	0.9	0.0
1000 quarters											
JPT	1.7	14.2	1.5	1.4	10.2	17.4	2.5	51.2	-	-	0.0
BGG	6.3	9.5	0.5	3.0	4.8	2.4	1.7	1.3	69.2	1.4	0.0
R^k Indexation	0.5	10.5	0.0	2.1	6.1	3.2	1.1	48.4	27.2	0.9	0.0

Observed spread

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	1.2	0.0	0.0	0.3	0.0	0.5	0.3	1.4	38.9	33.2	24.2
R^k Indexation	4.8	1.3	0.0	0.5	0.7	0.6	1.1	6.0	9.1	35.1	40.9
8 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	0.9	0.1	0.0	0.2	0.1	0.4	0.2	1.1	42.8	32.7	21.5
R^k Indexation	3.5	1.1	0.0	0.3	0.6	0.4	1.0	3.8	13.6	38.3	37.4
16 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	0.9	0.2	0.0	0.2	0.2	0.4	0.3	1.0	41.1	35.4	20.3
R^k Indexation	2.6	0.9	0.0	0.2	0.6	0.3	0.9	3.1	11.7	45.8	33.8
1000 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	0.9	0.2	0.0	0.2	0.2	0.5	0.3	1.0	39.4	37.7	19.5
R^k Indexation	2.6	0.9	0.0	0.2	0.6	0.3	0.9	3.1	11.7	45.8	33.8

The estimation results are summarized in Table 1. The BGG and R^k-indexation agency cost models easily dominate the JPT model as the JPT model cannot capture the forecastability of the risk premium that is in the data. Comparing BGG and R^k-indexation, the data rejects the BGG level of indexation preferring a level of contract indexation that is economically significant: $\chi_k = 2.24$ with a 90% confidence interval between 1.70 and 2.82. As suggested by the example in Section 2, indexation in excess of unity will imply significantly different responses to shocks compared to the BGG assumption. We will see this manifested in the IRF below.

Two other differences in parameter estimates are worth some comment. First, the BGG model estimates a significantly smaller size for investment adjustment costs (S'') in the table: $S'' = 1.09$ for BGG, but 2.43 for R^k-indexation, and 2.96 for JPT. The level of adjustment costs has two contrasting effects. First, lower adjustment costs will increase the response of investment to aggregate shocks. Second, lower adjustment costs imply smaller movements in the price of installed capital (Q_t) and thus smaller financial accelerator effects in the BGG model.

A second important difference in parameter estimates is in the standard deviation of the shocks. Compared to JPT, the BGG model estimates a significantly smaller volatility in the MEI shocks, and instead shifts this variance on to net worth shocks. Recall that the principal conclusion of JPT is the importance of the MEI shocks in the business cycle. But we once again end up with the JPT conclusion with regards to the importance of MEI shocks in the R^k-indexation model. An interesting question we take up below is why the BGG model downplays these shocks so significantly.

Table 2 reports the variance decomposition of three key variables: GDP, investment, and the risk premium. The JPT results are replicated here: the MEI shocks account for a substantial amount of business cycle variability in GDP (55% at the 4-quarter horizon) and investment (82% at the 4-quarter horizon). This conclusion is essentially unchanged with R^k-indexation. As anticipated by the example in Section 2, the estimated level of indexation results in real behavior similar to a model without agency costs. This is particularly clear in the IRFs presented in Fig. 3 that we discuss below.

In contrast to the R^k-indexation, BGG places much less weight on the MEI shocks and instead shifts this variance to the financial shocks (the idiosyncratic variance and net worth shocks) and the monetary policy shock. For the case of

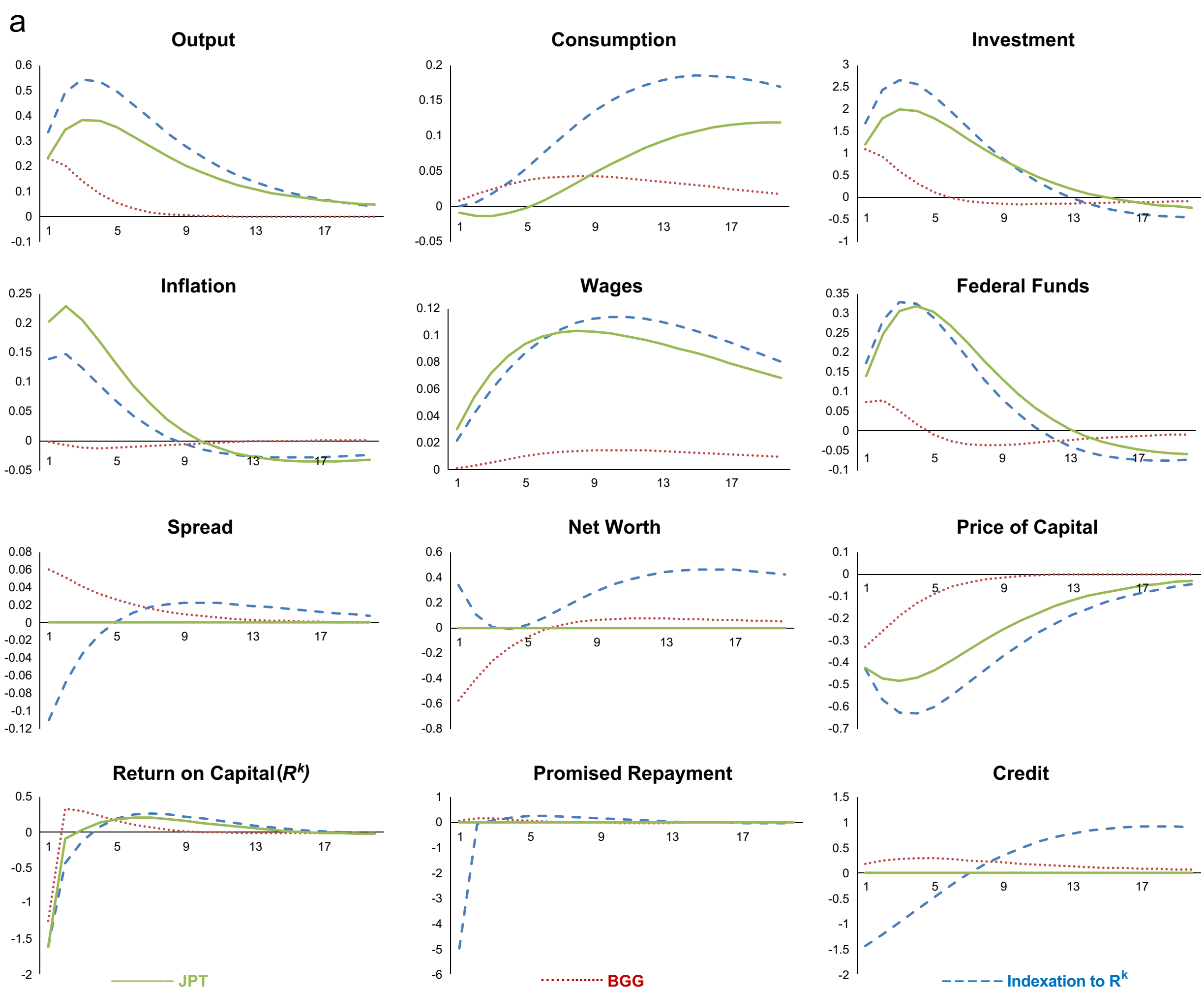

Fig. 3. (a) Impulse response functions to a one standard deviation marginal efficiency of investment shock using the estimated parameter values for each model and normalizing the standard deviation of the shock to the BGG model estimate.

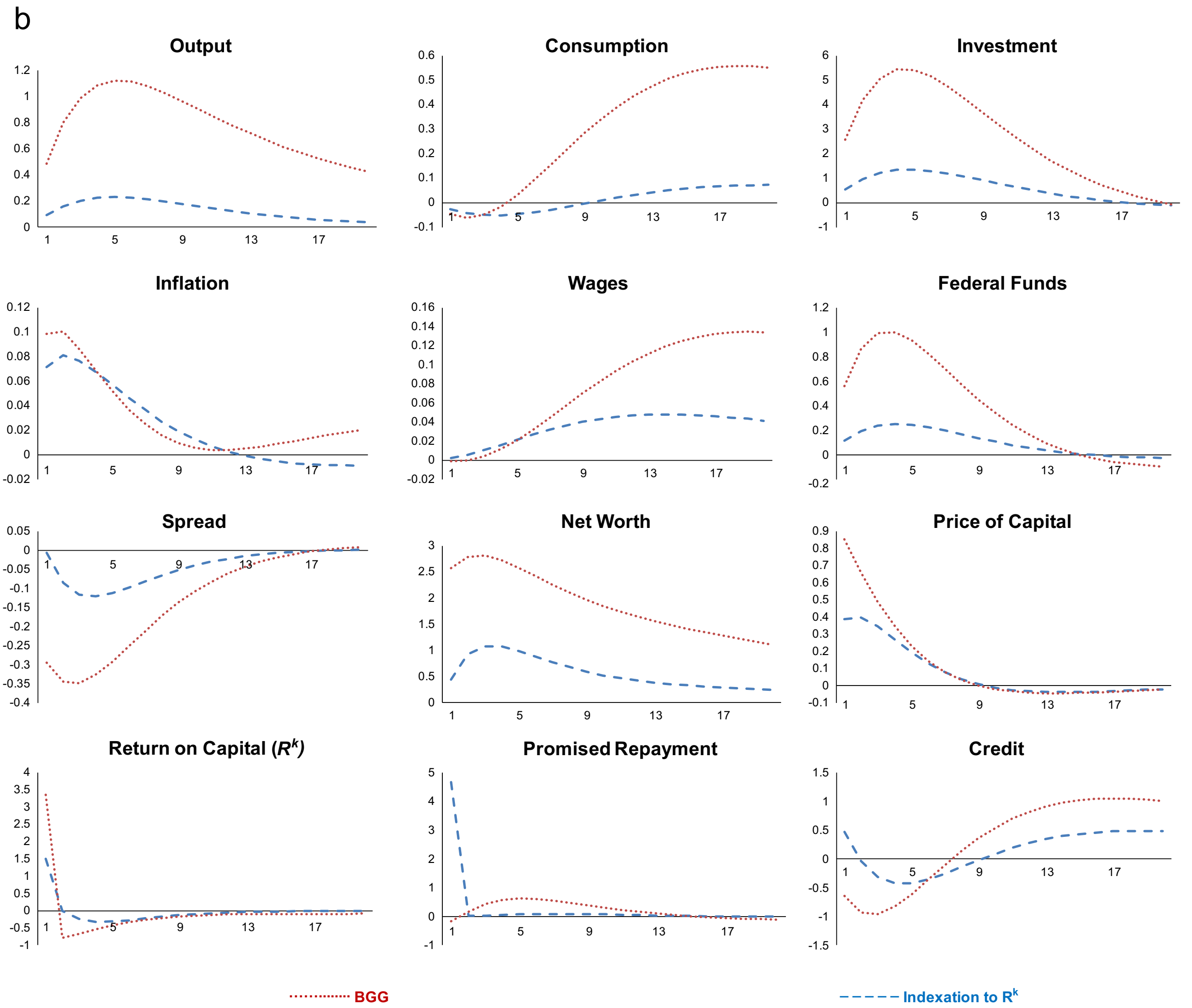

Fig. 3. (*continued*) (b) Impulse response functions to a one standard deviation net worth shock using the estimated parameter values for each model and normalizing the standard deviation of the shock to the BGG model estimate.

investment at the 4-quarter horizon, the BGG model places 4% of the variance on the MEI shocks (compared to 72% for the R^k-indexation model, and 82% for JPT). The importance of the two financial shocks increases from 20% under R^k-indexation, to 74% for BGG. The estimated level of financial shocks depends critically upon the estimated level of indexation.

The advantage of the financial models is showcased in the variance decomposition of the risk premium. By assumption, JPT assigns 100% of this variation to measurement error. In contrast, the financial models explain large portions of the risk premia movement by forces within the model. For example, at the 4-quarter horizon, the R^k indexation model assigns about 40% to measurement error, while this is only 24% for BGG. This predictability of the risk premium is echoed by DeGraeve (2008).

Why does the BGG model downplay the MEI shocks and thus shift variance to the other shocks? The answer is quite apparent from Fig. 3a. The figure sets all parameter values equal to those estimated for each model. To make the shocks comparable, the size of the impulse is set to the estimated SD under BGG. Returning to Fig. 3a, a positive innovation in MEI leads to a fall in the price of capital. Since the BGG contract is not indexed to the return to capital, the shock leads to a sharp decline in entrepreneurial net worth, and thus a sharp increase in the risk premium. This procyclical movement in the risk premium is in sharp contrast to the data. Hence, the Bayesian estimation in the BGG model estimates only a small amount of variability coming from the MEI shocks. Notice that in the R^k-indexation model net worth rises in response to an MEI shock, so that the impact effect on the risk premium is countercyclical. The main difference among models is the behavior of the repayment that under R^k-indexation is lowered in response to the drop in the return on capital, while in BGG it remains unchanged. The R^k-indexation model is thus consistent with MEI shocks driving the cycle, and the risk premium being countercyclical. The similarity of the R^k-indexation and JPT model is also apparent: the two IRFs to an MEI mirror each other

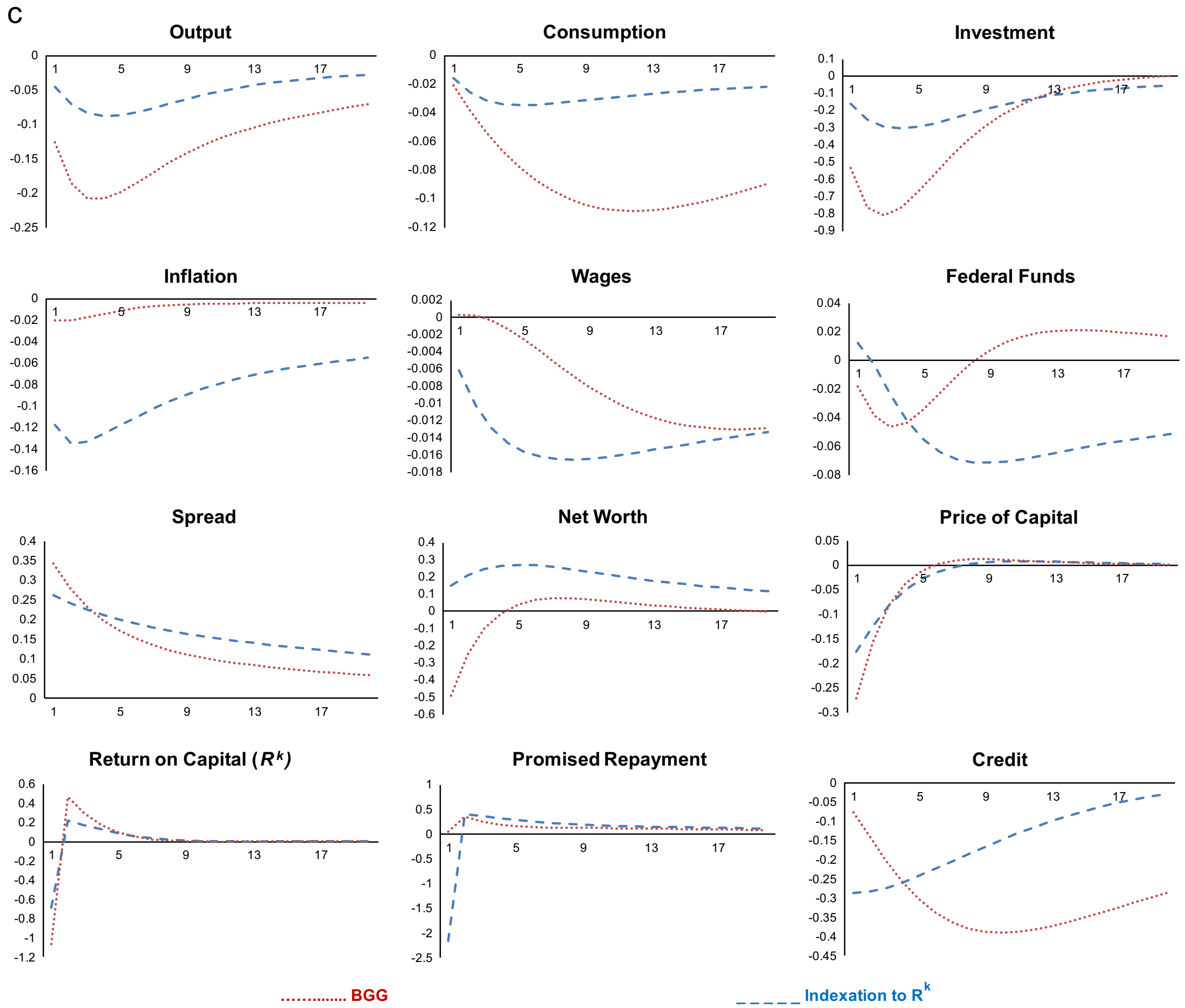

Fig. 3. (*continued*) (c) Impulse response functions to a one standard deviation idiosyncratic variance shock using the estimated parameter values for each model and normalizing the standard deviation of the shock to the BGG model estimate.

closely except that there is amplification in the agency cost model because of the decline in the risk premium (recall also that the IRFs are plotted for each model's estimated parameters).

Since the BGG model downplays the importance of MEI shocks, it must shift this variance to other shocks. Figs. 3b-3c plot the IRFs to the two financial shocks. Consider first the net worth shock. This shock has a larger effect on net worth and prices in the BGG model because of the multiplier mechanism outlined in Section 2. This effect is entirely absent in the indexation model. Further, the estimated level of nominal wage rigidity is higher under BGG so that production responds more sharply to this demand shock. There is similar amplification with the risk shock. The good news with the two financial shocks is that the spread is now countercyclical. The problem with the risk shock is that a fairly large movement in the spread has a very modest effect on real behavior, so that the estimation puts little weight on these shocks. As for net worth shocks, it is clear why the BGG model places such weight upon them: they lead to significant movement in output, co-movement among the aggregates, and a countercyclical risk premium. But for the indexation model, the lack of a financial accelerator means that these shocks have a fairly modest effect.

Fig. 3d plots the IRF to a monetary shock. In the case of BGG, the IRFs exhibit plausible co-movement and countercyclical spreads. The estimation does not put more weight on these policy shocks because the funds rate is an observable, and thus limits possible interest rate variability. One interesting and unique implication of the indexation model is that the risk premium is procyclical in response to monetary shocks. This conditional correlation is consistent with the evidence presented in Furlanetto et al. (2013). The reason BGG and the indexation model deliver different risk premium behavior has to do with the response of net worth to a monetary shock. The monetary contraction leads to a decline in investment demand (for the usual reasons) and a corresponding decline in the price of capital. Taken by itself, this decline in investment

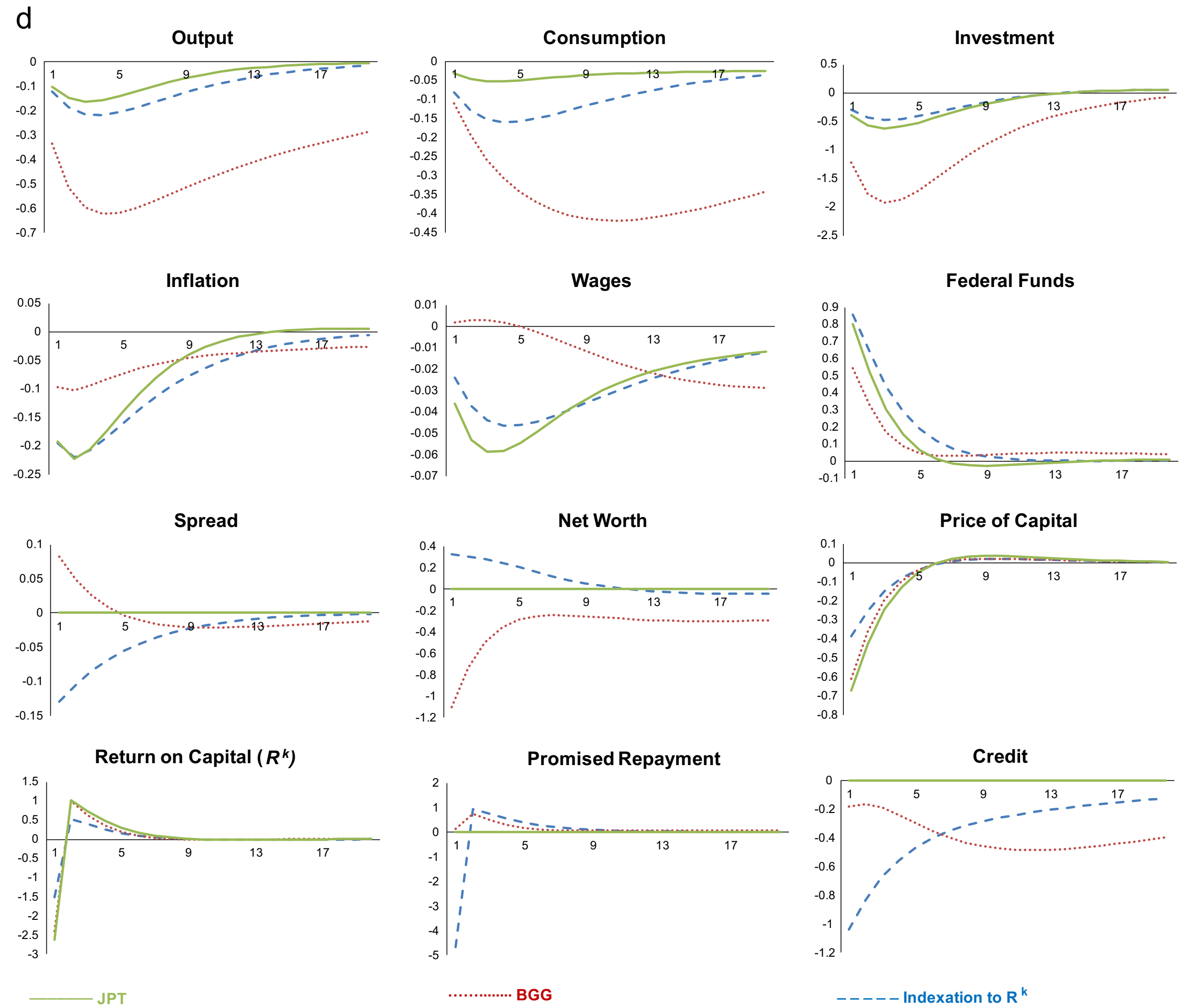

Fig. 3. (*continued*) (d) Impulse response functions to a one standard deviation monetary policy shock using the estimated parameter values for each model and normalizing the standard deviation of the shock to the BGG model estimate.

demand would lower the risk premium. But in BGG, this decline in the price of capital also leads to a sharp fall in net worth, and thus an increase in the risk premium. In contrast, the indexation model implies that the effect on net worth is largely sterilized as the repayment rate varies with the observed price of capital.

5. Sensitivity analysis

We have conducted a significant level of sensitivity analysis focusing on alternative observable variables, alternative steady-state calibrations, and an alternative definition of the risk premium. We summarize these results here. Our focus will be on the estimated level of indexation.

Table 3 presents a summary of the sensitivity analysis as we vary the financial variables used as observables. The series for leverage comes from Gilchrist et al. (2009). When net worth is treated as an observable, we follow Christiano et al. (2014) and link the model's growth rate of net worth with the growth rate in the Dow. In all cases we include serially correlated measurement error between the financial variables and their empirical counterparts. Note first that the indexation model dominates the BGG model from the measuring stick of log data density (JPT is always a distant third and is not reported). The basic conclusion from Table 3 is the robustness of the estimated level of indexation across the observables. Recall from Fig. 1, that there is a significant difference between an indexation level of 0 and 1, but very little quantitative difference in the financial accelerator as we move from, say, 1 to 2. Remarkably all the estimated indexation values imply essentially no financial accelerator. The final row of Table 3 returns to the baseline set of observables but imposes the monitoring cost value estimated by Christiano et al. (2014), $\mu_{mc}=0.21$. We hold the other primitives constant, so the steady state annual risk

Table 3
Sensitivity analysis. (Risk premium$=E_t\hat{r}^p_{t+1}-\hat{r}^d_t$).

	Estimated indexation[a]	BGG[b]	R^k indexation[b]
Baseline with model's risk premium defined as $E_t\hat{r}^p_{t+1}-\hat{r}^d_t$.	2.24 (1.70, 2.82)	−1383.1 0%	−1339.6 100%
Replace $\hat{r}^k_t$ with leverage as observable	2.14 (1.46, 2.65)	−1527.5 75%	−1534.4 25%
Replace $\hat{r}^k_t$ with net worth as an observable	0.90 (0.79, 1.01)	−1512.6 0%	−1491.6 100%
Add net worth as an observable	3.14 (2.80, 3.46)	−1862.9 0%	−1839.3 100%
Add leverage as an observable	2.96 (2.51, 3.46)	−1965.5 0%	−1875.6 100%
Baseline but with monitoring costs of $\mu_{mc}=0.21$	3.20 (2.88, 3.46)	−1368.5 0%	−1336.4 100%

Note: Unless otherwise noted, the financial observables include the risk premium and the return on capital series estimated by Gomme et al. (2011). The series for leverage comes from Gilchrist et al. (2009). When net worth is treated as an observable, we link the model's growth rate of net worth with the growth rate in the Dow. In all cases we include serially correlated measurement error between the financial variables and their empirical counterparts. The time period for the estimation is 1972:1–2008:4.

[a] In each cell, the first row is the point estimate, and the second row is the 90% confidence interval.

[b] In each cell, the first row is the log data density, and the second row is the posterior model probability.

Table 4
Sensitivity analysis. (Risk premium$=E_t\hat{r}^k_{t+1}-\hat{r}^d_t$).

	Estimated indexation[a]	BGG[b]	R^k indexation[b]
Baseline with risk premium defined as $E_t\hat{r}^k_{t+1}-\hat{r}^d_t$.	2.84 (2.37,3.32)	−1394.5 0%	−1348.3 100%
Replace $\hat{r}^k_t$ with leverage as observable	1.70 (1.36, 2.05)	−1555.8 0%	−1511.1 100%
Replace $\hat{r}^k_t$ with net worth as an observable	1.51 (1.20, 1.90)	−1586.4 0%	−1557.1 100%
Add net worth as an observable	2.40 (1.91, 2.88)	−1854.5 0%	−1818.8 100%
Add leverage as an observable	2.18 (1.84, 2.54)	−1922.6 0%	−1893.4 100%

Note: Unless otherwise noted, the financial observables include the risk premium and the return on capital series estimated by Gomme et al. (2011). The series for leverage comes from Gilchrist, Ortiz and Zakrajsek (2009). When net worth is treated as an observable, we link the model's growth rate of net worth with the growth rate in the Dow. In all cases we include serially correlated measurement error between the financial variables and their empirical counterparts. The time period for the estimation is 1972:1–2008:4.

[a] In each cell, the first row is the point estimate, and the second row is the 90% confidence interval.

[b] In each cell, the first row is the log data density, and the second row is the posterior model probability.

premium is now just 84 bp, and the external finance elasticity is $\nu=0.071$. As with the rest of the table, the estimation again implies no financial accelerator.

Table 4 presents the complementary evidence for an alternative measure of the risk premium, $(E_t\hat{r}^k_{t+1}-\hat{r}^d_t)$. This expression was used, for example, by DeGraeve (2008) in his comparison of the model's risk premium to its empirical counterpart. Note that this expression for the risk premium is, up to a linear approximation, proportional to the previous:

$$E_t\hat{r}^p_{t+1}-\hat{r}^d_t=\frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\nu\Theta_g(\kappa-1)}(E_t\hat{r}^k_{t+1}-\hat{r}^d_t-\hat{\sigma}_t) \tag{59}$$

This constant of proportionality is a fraction, equal to 0.23 for our baseline calibration. This alternative measure of the risk premium also implies a different calibration of the steady state. That is, to match a 200 bp annual spread of (R^k-R^d), a leverage ratio of $\overline{\kappa}=1.95$, and a quarterly default rate of 0.03/4, we need an entrepreneurial survival rate of $\gamma=0.98$, a standard deviation of $\sigma_{ss}=0.28$, and a monitoring cost of $\mu_{mc}=0.12$. This then implies that the agency cost elasticity is given by $\nu=0.041$. In any event, Table 4 presents the complementary set of results. The outcome is as in Table 3: the indexation model dominates, and the estimated level of indexation implies a trivial financial accelerator.

6. Conclusion

This paper began as an empirical investigation of the importance of agency costs and contract indexation in the business cycle. To reiterate, our principal results include the following. First, the financial models appear to be an improvement over the financial-frictionless JPT. Second, R^k-indexation appears to be an important characteristic of the data. Third, the importance of financial shocks (net worth and idiosyncratic variance) in explaining the business cycle is significantly affected by the estimated degree of R^k-indexation. In short, we find evidence for the importance of financial shocks in the business cycle. But the evidence also suggests that the effect of non-financial shocks on real activity is unaffected by the inclusion of financial forces in the model. That is, the results suggest the importance of financial shocks, but not the existence of a financial accelerator. This analysis thus implies that Bayesian estimation of financial models should include estimates of contract indexation. Empirical analyses that impose zero contract indexation likely distort both the source of business cycle shocks and their transmission mechanism.

Appendix A

A.1. Linearized system of equations

$$\hat{y}_t = \frac{y+F}{y}\left[\alpha \hat{k}_t + (1-\alpha)\hat{L}_t\right] \tag{A1}$$

$$\hat{\rho}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t \tag{A2}$$

$$\hat{s}_t = \alpha\hat{\rho}_t + (1-\alpha)\hat{w}_t \tag{A3}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\iota_p}E_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p}\hat{s}_t + \hat{\lambda}_{p,t} \tag{A4}$$

$$\begin{aligned}\hat{\lambda}_t =& \frac{h\beta e^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}E_t\hat{c}_{t+1} - \frac{e^{2\gamma_z}+h^2\beta}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_t + \frac{he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_{t-1} \\ &+ \frac{h\beta e^{\gamma_z}\rho_z - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{z}_t + \frac{e^{\gamma_z}-h\beta\rho_b}{(e^{\gamma_z}-h\beta)}\hat{b}_t + \left[\frac{h\beta e^{\gamma_z}\rho_\upsilon - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\right]\left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_t\end{aligned} \tag{A5}$$

$$\hat{\lambda}_t = \hat{R}_t + E_t\left(\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{\pi}_{t+1} - \left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_{t+1}\right) \tag{A6}$$

$$\hat{\rho}_t = \vartheta\hat{u}_t \tag{A7}$$

$$E_t\hat{r}^k_{t+1} = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{\upsilon}_{t+1} \tag{A8}$$

$$\hat{q}_t = -\hat{\mu}_t + e^{2(\gamma_z+\gamma_\upsilon)}S''\left(\hat{i}_t - \hat{i}_{t-1} + \hat{z}_t + \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t\right) - \beta e^{2(\gamma_z+\gamma_\upsilon)}S''E_t\left(\hat{i}_{t+1} - \hat{i}_t + \hat{z}_{t+1} + \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_{t+1}\right) \tag{A9}$$

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t \tag{A10}$$

$$\hat{\bar{k}}_t = (1-\delta)e^{-(\gamma_z+\gamma_\upsilon)}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t\right) + \left[1-(1-\delta)e^{-(\gamma_z+\gamma_\upsilon)}\right](\hat{\mu}_t + \hat{i}_t) \tag{A11}$$

$$\begin{aligned}\hat{w}_t =& \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta\iota_w)\xi_w}\hat{g}_{w,t} + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} \\ &+ \frac{\iota_w}{1+\beta}\left(\hat{z}_{t-1} + \left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_{t-1}\right) - \frac{1+\beta\iota_w-\beta\rho_z}{1+\beta}\hat{z}_t - \frac{1+\beta\iota_w-\beta\rho_\upsilon}{1+\beta}\left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_t + \hat{\lambda}_{w,t}\end{aligned} \tag{A12}$$

$$\hat{g}_{w,t} = \hat{w}_t - (\psi\hat{L}_t + \hat{b}_t - \hat{\lambda}_t) \tag{A13}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\phi_\pi\hat{\pi}_t + \phi_x(\hat{x}_t - \hat{x}^*_t)] + \phi_{dx}[(\hat{x}_t - \hat{x}_{t-1}) - (\hat{x}^*_t - \hat{x}^*_{t-1})] + \hat{\eta}_{mp,t} \tag{A14}$$

$$\hat{x}_t = \hat{y}_t - \frac{\rho k}{y}\hat{u}_t \tag{A15}$$

$$\frac{1}{g}\hat{y}_t = \frac{1}{g}\hat{g}_t + \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{\rho k}{y}\hat{u}_t \tag{A16}$$

$$\hat{r}^d_t = \hat{R}_t - E_t\hat{\pi}_{t+1} \tag{A17}$$

$$\hat{r}_t^k = \beta e^{-(\gamma_z+\gamma_\nu)}(1-\delta)\hat{q}_t + [1-\beta e^{-(\gamma_z+\gamma_\nu)}(1-\delta)]\hat{\rho}_t - \hat{q}_{t-1} \tag{A18}$$

For the **agency cost model**, we replace (A8) with

$$E_t\hat{r}_{t+1}^k = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{\upsilon}_{t+1} + \nu(\hat{q}_t + \hat{k}_t - \hat{n}_t) + \hat{\sigma}_t \tag{A8$'$}$$

And add the following equations:

$$\hat{n}_t = \kappa\frac{\gamma}{\beta}(\hat{r}_t^k - \hat{r}_t^l) + \frac{\gamma}{\beta}(\hat{r}_t^l + \hat{n}_{t-1}) + \gamma\kappa\frac{rp}{\beta}(\hat{k}_{t-1} + \hat{q}_{t-1} + \hat{r}_t^k) - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t + \hat{\eta}_{nw,t} \tag{A19}$$

$$\hat{r}_t^l = \hat{r}_{t-1}^d + [1+\theta_g(\chi_k - 1)](\hat{r}_t^k - E_{t-1}\hat{r}_t^k) \tag{A20}$$

A.2. The derivation of (A8) and (A20)

The optimal contract can be expressed as

$$V_{t+1}f'(\varpi_{t+1}) = \left[\frac{E_tV_{t+1}f'(\varpi_{t+1})}{E_t\Lambda_{t+1}g'(\varpi_{t+1})}\right]\Lambda_{t+1}g'(\varpi_{t+1}) \tag{A21}$$

$$\overline{\kappa}_tE_tR_{t+1}^kV_{t+1}f(\varpi_{t+1}) = \frac{-E_tV_{t+1}f'(\varpi_{t+1})}{E_t\Lambda_{t+1}g'(\varpi_{t+1})}R_t^dE_t\Lambda_{t+1} \tag{A22}$$

$$E_t\Lambda_{t+1}R_{t+1}^k\frac{\overline{\kappa}_t}{\overline{\kappa}_t - 1}g(\varpi_{t+1}) = R_t^dE_t\Lambda_{t+1} \tag{A23}$$

It is convenient to define $F(\varpi_{t+1}) \equiv ((-f'(\varpi_{t+1}))/(g'(\varpi_{t+1})))$, where $\Psi \equiv ((\varpi_{ss}F'(\varpi_{ss}))/(F(\varpi_{ss}))) > 0$, by the second order condition. Linearizing (A21)-(A23) we have

$$\Psi(\varpi_{t+1} - E_t\varpi_{t+1}) = (\lambda_{t+1} - E_t\lambda_{t+1}) + (v_{t+1} - E_tv_{t+1}) \tag{A24}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) + \kappa_t = (\Psi - \theta_f)E_t\varpi_{t+1} \tag{A25}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) = \left(\frac{1}{\kappa - 1}\right)\kappa_t - \theta_gE_t\varpi_{t+1} \tag{A26}$$

where $\theta_g \equiv ((\varpi_{ss}g'(\varpi_{ss}))/(g(\varpi_{ss})))$, with $0 < \theta_g < 1$, and $\theta_f \equiv ((\varpi_{ss}f'(\varpi_{ss}))/(f(\varpi_{ss}))) < 0$. Solving (A25) and (A26) we have:

$$E_t\varpi_{t+1} = \frac{\kappa}{\kappa - 1}\frac{1}{(\Psi - \theta_f + \theta_g)}\kappa_t \tag{A27}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) = \left[\frac{(\Psi - \theta_f + \theta_g) - \kappa\theta_g}{(\kappa - 1)(\Psi - \theta_f + \theta_g)}\right]\kappa_t \equiv \nu\kappa_t \tag{A28}$$

Using the definition of leverage and the deposit rate, (A28) is the same as (A8'). The linearized lender return and promised payment are given by:

$$\hat{r}_{t+1}^l = \hat{r}_{t+1}^k + \theta_g\varpi_{t+1} - \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{A29}$$

$$\hat{r}_{t+1}^p = \hat{r}_{t+1}^k + \varpi_{t+1} - \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{A30}$$

Combining (A24) and (A30) we have:

$$\hat{r}_t^p = r_{t-1}^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \frac{1}{\Psi}(\hat{\lambda}_t - E_{t-1}\hat{\lambda}_t) - \frac{1}{\Psi}(\hat{v}_t - E_{t-1}\hat{v}_t) \tag{A31}$$

where from (18) we have

$$\hat{v}_t = E_t\sum_{j=0}^{\infty}\beta^{j+1}(\Xi\hat{\kappa}_{t+j} + \hat{r}_{t+j+1}^k) \tag{A32}$$

where $\Xi \equiv [1 + (\beta/\gamma)(\nu - (1/(\kappa - 1)))]$. For parsimony we will estimate a promised payment of the form

$$\hat{r}_t^p = E_{t-1}\hat{r}_t^p + \chi_k(\hat{r}_t^k - E_{t-1}\hat{r}_t^k) \tag{A33}$$

Combining this with (A29) and (A30) we have:

$$\hat{r}_t^l = \hat{r}_{t-1}^d + [1+\theta_g(\chi_k - 1)](\hat{r}_t^k - E_{t-1}\hat{r}_t^k) \tag{A34}$$

This is just (A20).

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